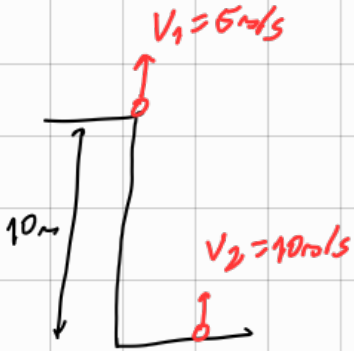


~ WEEK 1: 1D KINEMATICS ~

50013

A ball is thrown with an upward velocity of 5 m/s from the top of a 10 m building. One second later another ball is thrown vertically from the ground with an upward velocity of 10 m/s .

Determine the height from the ground where the balls pass each other.



①

$$x = x_0 + v \cdot (t - t_0) + \frac{1}{2} a (t - t_0)^2$$

$$x_1 = 10 + 5(t - 0) - \frac{1}{2} g (t - 0)^2$$

$$x_2 = 0 + 10(\underline{t - 1}) - \frac{1}{2} g (\underline{t - 1})^2$$

1 second later
substitute!!

② $x_1 = x_2$:

$$10 + 5t - \frac{1}{2} g t^2 = 10t - 10 - \frac{1}{2} g (t^2 - 2t + 1)$$

$$10 + 5t - \cancel{\frac{1}{2} g t^2} = 10t - 10 - \cancel{\frac{1}{2} g t^2} + g t - \frac{1}{2} g$$

$$20 + \frac{1}{2} g = 5t + g t$$

$$(g = 9.81) \rightarrow \underline{t = 1.682}$$

2022:

A point mass M with current position P moves along a straight line. The square of the velocity decreases linearly with x . The mass passes the location $x=0$ at $t=0$ with a velocity $v_0 > 0$ and at the location $x=x_1 > 0$ it has velocity $v_1=0$. At what time does the point mass reach the position x_1 and what is its acceleration.

① $t=0 \} V = v_0$
 $x=0$
 $x=x_1 \rightarrow V=0$
 $t \text{ for } x_1 = ?$
 $a = ?$

$$v = \frac{dx}{dt} \quad a = \frac{dv}{dt} \rightarrow dv = \frac{dv}{a}$$

$$\frac{1}{dt} = \frac{dv}{v} \rightarrow \frac{1}{dt} = \frac{v}{dx}$$

$$\frac{a}{dv} = \frac{v}{dx} \rightarrow a = v \cdot \frac{dv}{dx}$$

② $V^2 = C_1 x + C_2 \rightarrow V \text{ decreases linearly}$

$dx = v dt$

$dx = \sqrt{C_1 x + C_2} dt \rightarrow \frac{dx}{\sqrt{C_1 x + C_2}} = dt \rightarrow \int_0^x \frac{dx}{\sqrt{C_1 x + C_2}} = \int_0^t dt$

$= \frac{1}{C_1} \int u^{-1/2} du = \frac{2}{C_1} u^{1/2}$

$C_1 x + C_2 = u$
 $C_1 dx = du$

③ $x=0 \rightarrow v=v_0 \rightarrow v_0^2 = C_2$
 $x=x_1 \rightarrow v=0 \rightarrow 0 = C_1 x_1 + C_2$

$(v^2 = C_1 x + C_2)$

$C_1 = \frac{v_0^2}{x_1}$
 $v^2 = v_0^2 \left(1 - \frac{x}{x_1}\right)$

④ $= \frac{-2x_1}{v_0^2} \cdot v_0 \cdot \sqrt{1 - \frac{x}{x_1}} = \frac{-2x_1}{v_0} \cdot \sqrt{1 - \frac{x}{x_1}} = t \Big|_0^x = \frac{2x_1}{v_0} \left(\sqrt{1 - \frac{x}{x_1}} + 1 \right) = t$

⑤ $x=x_1 \rightarrow t = \frac{2x_1}{v_0}$

⑥

$a = ?$
 $dv = a dt \rightarrow a = \frac{dv}{dt} = v \frac{dv}{dx}$

$\frac{d}{dx} \left(\frac{1}{\sqrt{1 - \frac{x}{x_1}}} \right) = \frac{1}{2} \cdot \frac{1}{1 - \frac{x}{x_1}} \cdot \frac{1}{x_1}$

$a = \left(v_0 \sqrt{1 - \frac{x}{x_1}} \right) \cdot \left(v_0 \cdot \frac{-1}{2x_1} \cdot \frac{1}{\sqrt{1 - \frac{x}{x_1}}} \right) = \left[-\frac{1}{2} \cdot \frac{v_0^2}{x_1} \right]$

Not:

$v^2 = C_1 x + C_2 \rightarrow \frac{d}{dt} 2v \frac{dv}{dt} = C_1 \frac{dx}{dt} \rightarrow 2va = C_1 x \rightarrow a = \frac{C_1}{2} = \frac{-v_0^2}{2x_1}$

Schlechte die neue Notation.

Exp 3

Acceleration of a particle moving along a straight line is defined by $a = (2t-9) \text{ m/s}^2$ where t is in seconds.

If $x(0) = 1 \text{ m}$ and $v(0) = 10 \text{ m/s}$ determine the particles position, velocity and the total distance travelled when $t = 9 \text{ s}$.

①

$$a = 2t - 9$$

$$x(0) = 1 \text{ m}$$

$$v(0) = 10 \text{ m/s}$$

$$a = \int u(t) dt$$

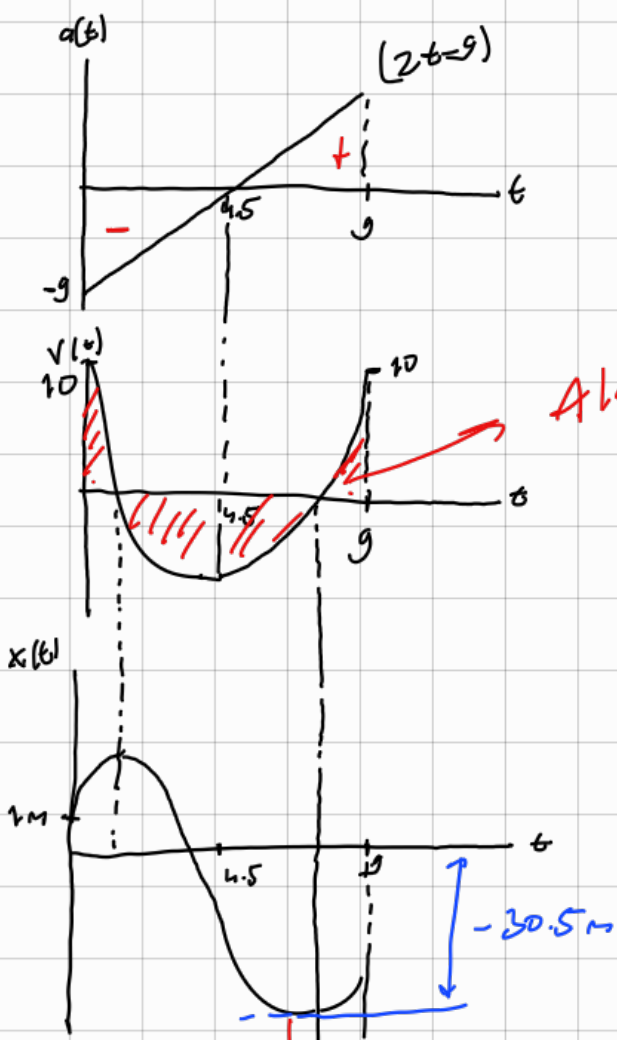
$$u(0) = 10$$

$$C_1 = 10$$

$$= \frac{2t^2}{2} - 9t + C_1$$

$$v(t) = t^2 - 9t + 10$$

$$v(t) = \int a dt$$



Absolut topları = Toplam yer değişimi

$$= 56 \text{ m}$$

$$v(t) = t^2 - 9t + 10 = 0$$

→ kökler x'in ekstrem noktaları verir.

9'a eşit yavaşlar

$$x(t) = \int v dt = 1 + 10t + \frac{t^2}{2} - \frac{9t^2}{2} = -30.5 \text{ m}$$

$$\Delta x = \int_0^{t_1} v dt + \left| \int_{t_1}^{t_2} v dt \right| + \int_{t_2}^9 v dt = 56 \text{ m}$$

negatif olduğundan mutlak değer

Sample A small projectile is fired vertically downward into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3) \text{ m/s}^2$, where v is in m/s.

Determine the projectile's velocity and position 4 s after it is fired.

A small projectile is fired vertically downward into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3) \text{ m/s}^2$, where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.

SOLUTION

Coordinate System. Since the motion is downward, the position coordinate is positive downward, with origin located at O , Fig. 12-3.

Velocity. Here $a = f(v)$ and so we must determine the velocity as a function of time using $a = dv/dt$, since this equation relates v , a , and t . (Why not use $v = v_0 + a_c t$?) Separating the variables and integrating, with $v_0 = 60 \text{ m/s}$ when $t = 0$, yields

$$\begin{aligned}
 (+\downarrow) \quad a &= \frac{dv}{dt} = -0.4v^3 \\
 \int_{60}^v \frac{dv}{-0.4v^3} &= \int_0^t dt \\
 \frac{1}{-0.4} \left(\frac{1}{-2} \right) \frac{1}{v^2} \bigg|_{60}^v &= t - 0 \\
 \frac{1}{0.8} \left[\frac{1}{v^2} - \frac{1}{(60)^2} \right] &= t \\
 v &= \left\{ \left[\frac{1}{(60)^2} + 0.8t \right]^{-1/2} \right\} \text{ m/s}
 \end{aligned}$$

Here the positive root is taken, since the projectile will continue to move downward. When $t = 4 \text{ s}$,

$$v = 0.559 \text{ m/s} \downarrow$$

Ans.

Position. Knowing $v = f(t)$, we can obtain the projectile's position from $v = ds/dt$, since this equation relates s , v , and t . Using the initial condition $s = 0$, when $t = 0$, we have

$$\begin{aligned}
 (+\downarrow) \quad v &= \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t \right]^{-1/2} \\
 \int_0^s ds &= \int_0^t \left[\frac{1}{(60)^2} + 0.8t \right]^{-1/2} dt \\
 s &= \frac{2}{0.8} \left[\frac{1}{(60)^2} + 0.8t \right]^{1/2} \bigg|_0^t \\
 s &= \frac{1}{0.4} \left\{ \left[\frac{1}{(60)^2} + 0.8t \right]^{1/2} - \frac{1}{60} \right\} \text{ m}
 \end{aligned}$$

When $t = 4 \text{ s}$,

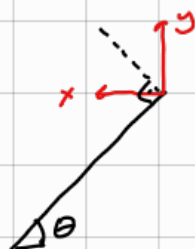
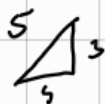
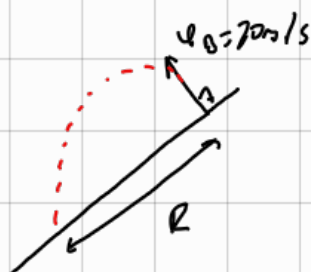
$$s = 4.43 \text{ m}$$

Ans.



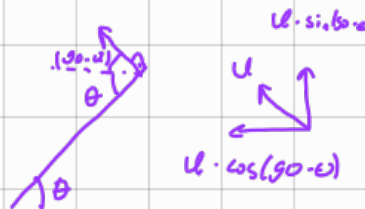
Fig. 12-3

Ex 12 Water is sprayed 90° from the slope at 20 m/s . Determine the range R .



$$x = u_{0x} \cdot t = R \cdot \cos \theta$$

$$y = u_{0y} \cdot t - \frac{1}{2} g t^2 = -R \cdot \sin \theta$$



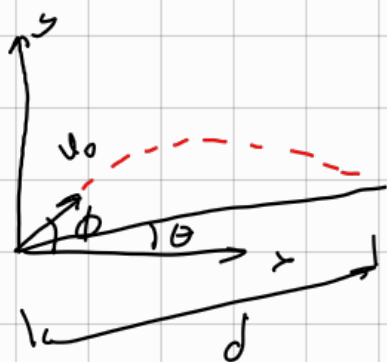
$$u_0 \cdot \sin \theta \cdot t = R \cdot \cos \theta \quad \rightarrow \quad R = \frac{u_0 \cdot \sin \theta}{\cos \theta} \cdot t$$

$$u_0 \cdot \cos \theta \cdot t - \frac{1}{2} g t^2 = -R \sin \theta \quad \xrightarrow{\text{bring over}} \quad u_0 \cdot \cos \theta \cdot t - \frac{1}{2} g t^2 = -u_0 \cdot \frac{\sin \theta}{\cos \theta} \cdot t \cdot \sin \theta$$

$$t = \frac{2}{g} \frac{1}{\cos \theta} u_0$$

$$R = u_0 \cdot \frac{\sin \theta}{\cos \theta} \cdot \frac{2}{g} \frac{1}{\cos \theta} u_0$$

Ex 2 For d to be max, determine ϕ , θ const.



$$x: v_0 \cdot \cos \phi \cdot t = d \cdot \cos \theta \quad (1)$$

$$y: v_0 \cdot \sin \phi \cdot t - \frac{1}{2} g t^2 = d \sin \theta \quad (2)$$

$$(1): t = \frac{d}{v_0} \cdot \frac{\cos \theta}{\cos \phi} \quad \text{sub into (2)}$$

$$(2): \cancel{v_0} \cdot \sin \phi \cdot \frac{d}{\cancel{v_0}} \cdot \frac{\cos \theta}{\cos \phi} - \frac{1}{2} g \frac{d^2}{v_0^2} \cdot \frac{\cos^2 \theta}{\cos^2 \phi} = \cancel{d} \sin \theta$$

$$d = \frac{2 \cdot v_0^2 \cdot \cos \theta}{g \cos^2 \theta} \cdot \underbrace{(\cos \theta \cdot \sin \phi - \sin \theta \cdot \cos \phi)}_{\sin(\phi - \theta)}$$

$$\frac{d(d)}{d\phi} = 0 \rightarrow -\sin \phi \cdot \sin(\phi - \theta) + \cos \phi \cdot \cos(\phi - \theta) = 0$$

extremum

$$\boxed{\tan \phi \cdot \tan(\phi - \theta) = 1}$$

$$\theta = 0 \quad \text{if } \phi = 45^\circ \quad \text{optimal! (check it!)} \quad \checkmark$$

$$\sin \phi \cdot \frac{\cos \theta}{\cos \phi} - \frac{1}{2} g \frac{d}{v_0^2} \cdot \frac{\cos^2 \theta}{\cos^2 \phi} = \sin \theta$$

$$\frac{1}{2} g \frac{d}{v_0^2} \cdot \frac{\cos^2 \theta}{\cos^2 \phi} = \sin \phi \cdot \frac{\cos \theta}{\cos \phi} - \sin \theta$$

$$d = \left(\sin \phi \cdot \frac{\cos \theta}{\cos \phi} - \sin \theta \right) \cdot \frac{\cos^2 \phi}{\cos^2 \theta} \cdot \frac{v_0^2 \cdot 2}{g}$$

$$d = (\sin \phi \cdot \cos \theta - \sin \theta \cdot \cos \phi) \cdot \frac{\cos \phi}{\cos^2 \theta} \cdot \frac{v_0^2 \cdot 2}{g}$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cdot \cos \beta \pm \sin \beta \cdot \cos \alpha$$

$$\sin' \rightarrow \cos$$

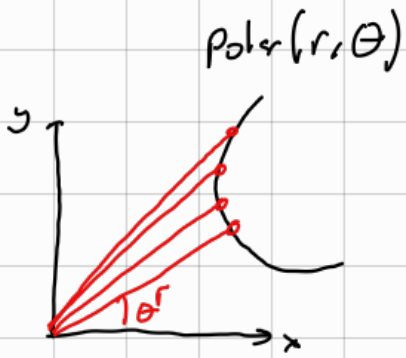
$$\cos' \rightarrow -\sin$$

$$(\vec{v}_1 \cdot \vec{v}_2)' = \vec{v}_1 \cdot \vec{g}_1 + g_1 \cdot \vec{v}_1$$

$$\left(\frac{2 \cdot v_0^2 \cdot \cos \theta}{g \cdot \cos^2 \theta} \cdot \sin(\phi - \theta) \right)' = \frac{2 v_0^2}{g \cdot \cos^2 \theta} \cdot (-\sin \phi) \sin(\phi - \theta) + \cos(\phi - \theta) \cdot \frac{2 v_0^2}{g \cdot \cos^2 \theta} \cdot \cos \phi$$

$$\frac{2 v_0^2}{g \cdot \cos^2 \theta} \left((-\sin \phi) \cdot \sin(\phi - \theta) + \cos \phi \cdot \cos(\phi - \theta) \right) = 0$$

~ Polar Eksen Takımı ~ eksen doğrultuları değişken



$$\vec{r} = r \cdot \vec{e}_r \rightarrow \text{konu}$$

$$\vec{v} = \dot{\vec{r}} = \dot{r} \cdot \vec{e}_r + r \cdot \dot{\theta} \cdot \vec{e}_\theta \rightarrow \text{Hiz}$$

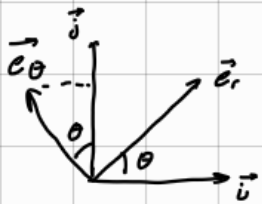
$$= \underbrace{\dot{r}}_{u_r} \cdot \vec{e}_r + r \underbrace{\dot{\theta}}_{u_\theta} \cdot \vec{e}_\theta$$

$$= u_r \cdot \vec{e}_r + u_\theta \cdot \vec{e}_\theta$$

$$\left. \begin{aligned} \vec{r} &= x \vec{i} + y \vec{j} \\ \vec{v} &= u_x \vec{i} + u_y \vec{j} \\ \vec{a} &= a_x \vec{i} + a_y \vec{j} \end{aligned} \right\} \text{Dik koordinat}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = (\ddot{r} \vec{e}_r + r \ddot{\theta} \vec{e}_\theta) + (\dot{r} \dot{\theta} \vec{e}_\theta + r \ddot{\theta} \vec{e}_\theta + r \dot{\theta} \dot{\theta} \vec{e}_r) =$$

$$\vec{a} = \underbrace{(\ddot{r} - r \dot{\theta}^2)}_{a_r} \vec{e}_r + \underbrace{(2\dot{r} \dot{\theta} + r \ddot{\theta})}_{a_\theta} \vec{e}_\theta \rightarrow \text{ivme}$$



$$\vec{e}_r = \cos \theta \vec{i} + \sin \theta \vec{j}$$

$$\dot{\vec{e}}_r = -\sin \theta \cdot \dot{\theta} \vec{i} + \cos \theta \cdot \dot{\theta} \vec{j} = (-\sin \theta \vec{i} + \cos \theta \vec{j}) \cdot \dot{\theta}$$

$\left(\frac{d}{dt} = \frac{d}{d\theta} \cdot \frac{d\theta}{dt} \right)$

angular velocity (rad/s)
(açılma hızı)

$\dot{\theta} \rightarrow$ angular velocity
 $\ddot{\theta} \rightarrow$ angular acceleration

$$\vec{e}_\theta = -\sin \theta \vec{j} + \cos \theta \vec{i} \rightarrow \vec{e}_r = \vec{e}_\theta \cdot \dot{\theta}$$

$$\dot{\vec{e}}_\theta = -\dot{\theta} \cdot \vec{e}_r$$

Tehtedeki özet

$$\begin{aligned} u_\theta &= r \dot{\theta} \\ u_r &= \dot{r} \\ a_\theta &= 2\dot{r} \dot{\theta} + r \ddot{\theta} \\ a_r &= \ddot{r} - r \dot{\theta}^2 \end{aligned}$$

Polar (r, theta)

angular acceleration
angular velocity (rad/s)

$\vec{r} = r \vec{e}_r$
 $\vec{v} = \dot{\vec{r}} = \dot{r} \vec{e}_r + r \dot{\theta} \vec{e}_\theta$
 $\vec{a} = \frac{d\vec{v}}{dt} = (\ddot{r} - r \dot{\theta}^2) \vec{e}_r + (2\dot{r} \dot{\theta} + r \ddot{\theta}) \vec{e}_\theta$

$\vec{e}_r = \cos \theta \vec{i} + \sin \theta \vec{j}$
 $\vec{e}_\theta = -\sin \theta \vec{i} + \cos \theta \vec{j}$
 $\dot{\vec{e}}_r = -\sin \theta \dot{\theta} \vec{i} + \cos \theta \dot{\theta} \vec{j} = \dot{\theta} (-\sin \theta \vec{i} + \cos \theta \vec{j}) = \dot{\theta} \vec{e}_\theta$
 $\dot{\vec{e}}_\theta = -\cos \theta \dot{\theta} \vec{i} - \sin \theta \dot{\theta} \vec{j} = -\dot{\theta} (\cos \theta \vec{i} + \sin \theta \vec{j}) = -\dot{\theta} \vec{e}_r$

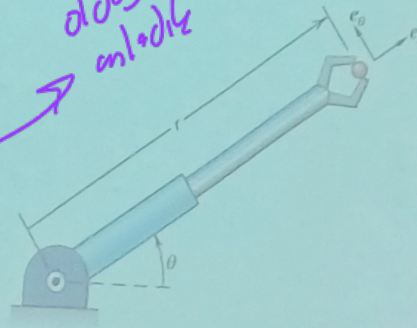
$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} (\dot{r} \vec{e}_r + r \dot{\theta} \vec{e}_\theta) = (\ddot{r} \vec{e}_r + \dot{r} \dot{\theta} \vec{e}_\theta) + (\dot{r} \dot{\theta} \vec{e}_\theta + r \ddot{\theta} \vec{e}_\theta + r \dot{\theta} \dot{\theta} \vec{e}_r) = (\ddot{r} - r \dot{\theta}^2) \vec{e}_r + (2\dot{r} \dot{\theta} + r \ddot{\theta}) \vec{e}_\theta$

Exp 1

The length of a robotic arm to its end effector varies according to $r(\theta) = 0.5 + 0.25(1 - \cos 2\theta)$.

Suppose the robotic arm is rotating at a constant angular speed of 0.5 rad/s.

Determine the magnitude of the velocity and acceleration for the robotic arm's end effector when $\theta = 45^\circ$.



Polar koordinatlar

$$r = 0.5 + 0.25(1 - \cos 2\theta), \quad \theta = 45^\circ \left(\frac{\pi}{4}\right)$$

$$|\vec{v}|, |\vec{a}| = ?$$

$$\begin{aligned} & (0.5 \cdot \sin 2\theta \cdot \dot{\theta})^2 \\ & 0.5 \cdot \cos 2\theta \cdot 2 \cdot \dot{\theta} + \ddot{\theta} \cdot 0.5 \cdot \sin 2\theta \\ & = \cos 2\theta \cdot \dot{\theta} + 0.5 \cdot \sin 2\theta \cdot \ddot{\theta} \end{aligned}$$

$$\begin{aligned} \vec{v} &= \dot{r} \cdot \vec{e}_r + r \dot{\theta} \cdot \vec{e}_\theta = 0.25 \cdot 2 \cdot \sin 2\theta \cdot \dot{\theta} \cdot \vec{e}_r + r \cdot \dot{\theta} \cdot \vec{e}_\theta \\ &= 0.5 \cdot 0.5 \cdot \vec{e}_r + 0.75 \cdot 0.5 \cdot \vec{e}_\theta \end{aligned}$$

angular speed = 0.5 rad/s

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2) \vec{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta}) \vec{e}_\theta$$

$$\ddot{r} = \cos 2\theta \ddot{\theta} + \dot{\theta} \cdot 0.5 \cdot \sin 2\theta = 0$$

$$\dot{\theta} = 0.5 \text{ rad/s}$$

$$\ddot{\theta} = 0 \text{ rad/s}^2$$

$$a_r = -0.75 \cdot 0.5^2$$

$$a_\theta = 2 \cdot 0.5^2 \cdot 0.5$$

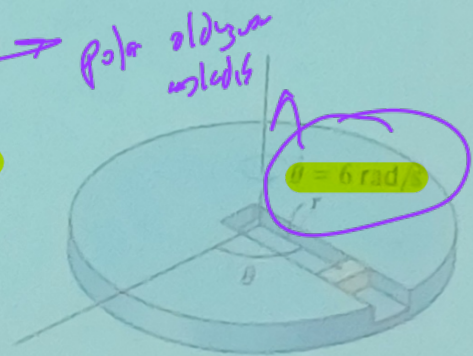
$$|\vec{a}| = \sqrt{a_r^2 + a_\theta^2} = \dots$$

$$|\vec{v}| = \sqrt{v_r^2 + v_\theta^2} = \dots$$

Ex 2

A block moves outward along the slot in the platform with a speed of $(dr/dt) = 4t$. The platform rotates at a constant rate of 6 rad/s.

If the block starts from rest at the center, determine the magnitudes of its velocity and acceleration when $t = 1$ s.



$$\vec{v} = \dot{r} \cdot \vec{e}_r + r \dot{\theta} \cdot \vec{e}_\theta$$

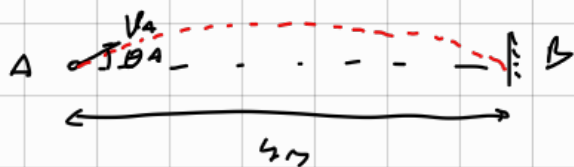
$$= 4 \cdot 1 \cdot \vec{e}_r + 2 \cdot 1^2 \cdot 6 \cdot \vec{e}_\theta$$

$\dot{r} = 4t \rightarrow r = \int 4t dt = 2t^2$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2) \vec{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta}) \vec{e}_\theta$$

$$= (4 - 2 \cdot 6^2) \vec{e}_r + (2 \cdot 2 \cdot 1 \cdot 6 + 2 \cdot 0) \vec{e}_\theta$$

Ex 3 A dot is thrown with a speed of 10 m/s. Determine the longest possible time when it strikes the target, the corresponding angle θ_A at which it should be thrown, and the velocity of the dot when it strikes the target.



Max/min soruları her zaman ekstremum problemi olduğuna göre türev kullanılır.

$$v_A = 10 \text{ m/s}$$

$$x = 4 \text{ m}$$

$$x: 10 \cdot \cos \theta \cdot t = 4 \text{ m} \quad (2) \rightarrow x = v_{Ax} \cdot t$$

$$y: 10 \cdot \sin \theta \cdot t - \frac{1}{2} g t^2 = 0 \rightarrow 10 \cdot \sin \theta \cdot t = \frac{1}{2} g t^2 \quad (1)$$

$$(1)^2 + (2)^2 = 10^2 \cdot t^2 = 4^2 + \left(\frac{1}{2} g t^2\right)^2$$

$$t^2 = u \rightarrow \frac{1}{4} g^2 u^2 - 100u + 4^2 = 0$$

$$\Delta = \frac{-100 \pm \sqrt{10^4 - g^2}}{g^2} > 0$$

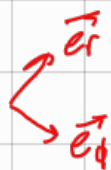
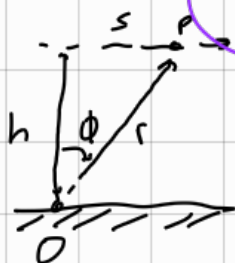
$$\begin{cases} t_1 = \dots \\ t_2 = \dots \end{cases}$$

büyük olanı al
çünkü bu

$$10 \cdot \cos \theta \cdot t = 4 \rightarrow \theta = \dots$$

Exp 4

An observer watches the flight of a low flyer P, flying with const. speed U_0 in a flight altitude h . Calculate the angular acceleration of his head and the radial acceleration (in terms of U_0, h, ϕ)

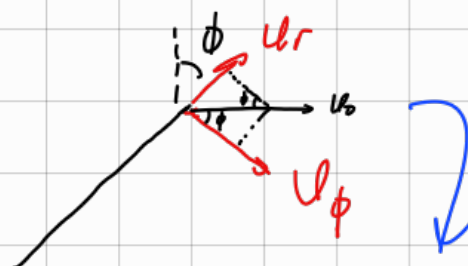


$\ddot{\phi} = ?$ → angular acceleration
 $\ddot{r} = ?$ → radial acceleration

$$\vec{U} = \dot{r} \cdot \vec{e}_r + r \cdot \dot{\phi} \vec{e}_\phi = U_0 \vec{e}_\phi$$

$$\vec{a} = (\ddot{r} - r\dot{\phi}^2) \vec{e}_r + (2\dot{r}\dot{\phi} + r\ddot{\phi}) \vec{e}_\phi = 0$$

const. speed (U_0)



$$U_r = U_0 \sin \phi$$

$$U_\phi = U_0 \cos \phi$$

$$r \dot{\phi} = U_\phi \Rightarrow \dot{\phi} = \frac{U_0}{h} \cdot \cos^2 \phi$$

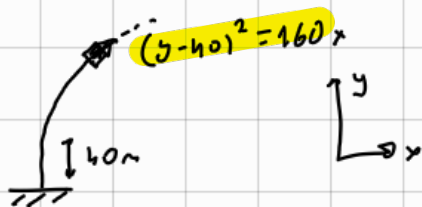
$$\ddot{\phi} = \frac{2U_0}{h} \cdot \cos \phi \cdot \sin \phi \cdot \dot{\phi}$$

$$= -\frac{U_0}{h} \cdot \sin 2\phi \dot{\phi}$$

$$\ddot{\phi} = -\left(\frac{U_0}{h}\right)^2 \cdot \sin 2\phi \cdot \cos^2 \phi$$

Exp 5

Velocity component in the vertical direction is const. as 180 m/s. Determine the magnitudes of the rocket's velocity and acceleration when it reaches an altitude of 80 m.



$$\frac{(y-40)^2}{160} = x$$

$$\frac{d}{dt} \rightarrow \frac{2(y-40) \cdot \dot{y}}{160} = \dot{x} = U_x$$

$$y=80 \rightarrow U_x = \frac{80 \cdot 180}{160}$$

$$\dot{y} = 180 \text{ (const.)}$$

↳ U_y

$$\frac{d}{dt} \rightarrow \frac{2\dot{y}^2 + 2(y-40)\ddot{y}}{160} = \ddot{x} = a_x$$

$$y=80 \rightarrow a_x = \dots$$

Ex 6. Co travels around the circular track with a const. speed of 20 m/s.

Determine the car's radial and transverse components of velocity and acceleration for $\theta = \frac{\pi}{4}$ rad.



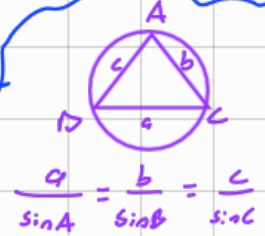
$$r = (400 \cos \theta) \text{ m}$$

$$\frac{r'}{\sin \theta} = \frac{r}{\sin(180-2\theta)}$$

obtain radial velocity

$$\sin 2\theta$$

(sin theorem)



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

(1)



$$r' = \frac{r}{\sin 2\theta} \cdot \sin \theta$$

$$r' = \frac{400 \cdot \cos \theta}{2 \cdot \sin \theta \cos \theta} \cdot \sin \theta = \underline{200 \text{ m/s}}$$

r sbl. trans 0

$$a_\theta = 2r'\dot{\theta} + r\ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

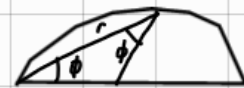
hit sbl. 2. trans 0 obt.

$$v = v_0 = r \cdot \dot{\theta}, \quad \dot{\theta} = \frac{20}{200} = \frac{1}{10}$$

$$\begin{aligned} \sin(90-\alpha) &= \cos \alpha \\ \cos(90-\alpha) &= \sin \alpha \\ \sin(180-\alpha) &= \sin \alpha \\ \cos(180-\alpha) &= -\cos \alpha \end{aligned}$$

obtain transverse velocity

(2)



$$r = 400 \cos \phi$$

$$\dot{\phi} = \frac{\dot{\theta}}{2}$$

$$\dot{r} = -400 \sin \phi \cdot \dot{\phi}$$

$$\ddot{r} = -400 \cos \phi \cdot \dot{\phi}^2 \cdot \ddot{\phi}$$

$$a_\theta = 2\dot{r}\dot{\phi} + r\ddot{\phi}$$

$$a_r = \ddot{r} - r\dot{\phi}^2$$

(Juan's son's desk)

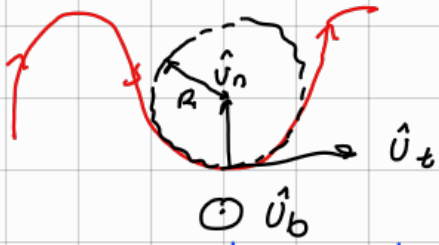
~ WEEK 3 ~

MEKA
KINEMATİK

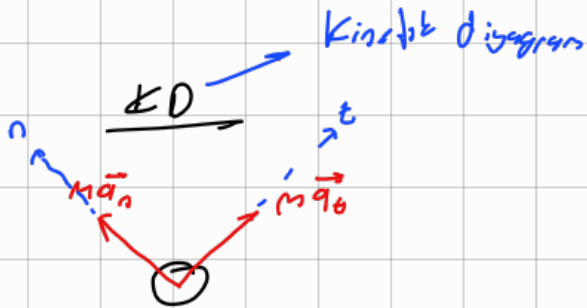
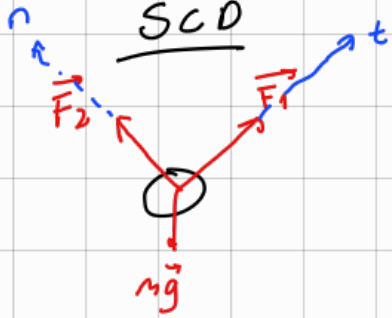
~ KINETICS N-2 COORDINATES ~

Sonrada sağlandı
(ders kayıldı)

$$\vec{a} = \dot{v} \cdot \hat{u}_t + \frac{v^2}{R} \hat{u}_n$$



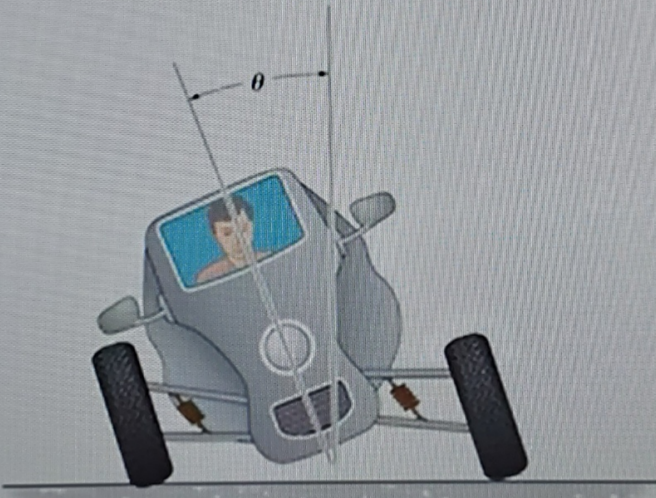
$\odot \hat{u}_b$
sağ. duvarında dışı
(içinde giriyob)

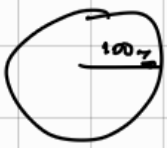


$$\sum F_n = m \cdot a_n = \frac{m v^2}{R}$$

$$\sum F_t = m \cdot a_t = m \cdot \dot{v}$$

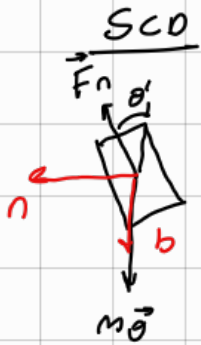
13-65. The vehicle is designed to combine the feel of a motorcycle with the comfort and safety of an automobile. If the vehicle is traveling at a constant speed of 80 km/h along a circular curved road of radius 100 m, determine the tilt angle θ of the vehicle so that only a normal force from the seat acts on the driver. Neglect the size of the driver.





$$\vec{a} = \cancel{\dot{\theta} \cdot \hat{u}_t} + \frac{v^2}{R} \hat{u}_n \rightarrow \underline{a_n = \frac{v^2}{R}}$$

const. speed



$\odot_t \rightarrow$ daireye $\curvearrowright$ dândırık, qaba kızıltın dâim dâim salıyır.
(şekildedeki gibi)

KD



$\odot_t$

$$\sum F_b = 0 = F_n \cos \theta - mg \rightarrow \frac{mg}{F_n} = \cos \theta \quad (1)$$

$$\sum F_n = m \cdot a_n = m \cdot 80^2 \cdot \frac{kn^2}{h^2} \cdot \left(\frac{1000m}{1kn} \right)^2 \cdot \frac{1h}{3600s} =$$

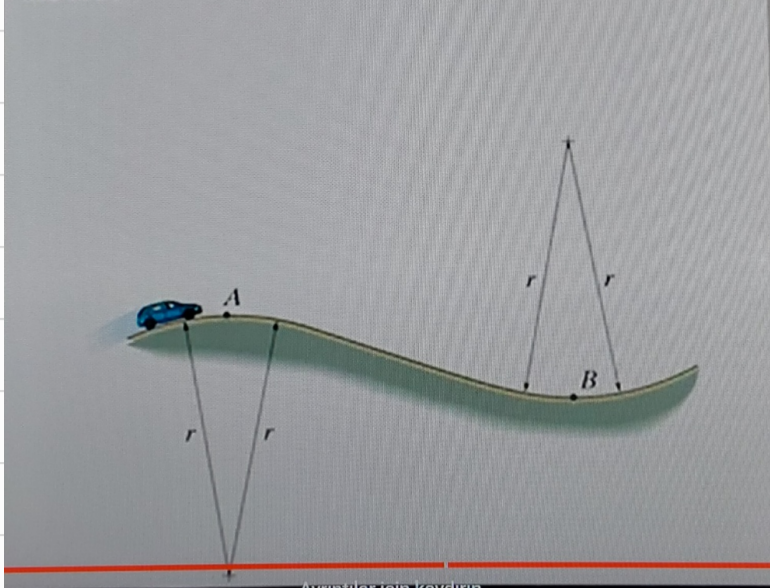
$$= 4.938m = F_n \sin \theta$$

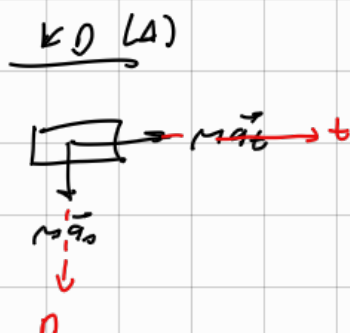
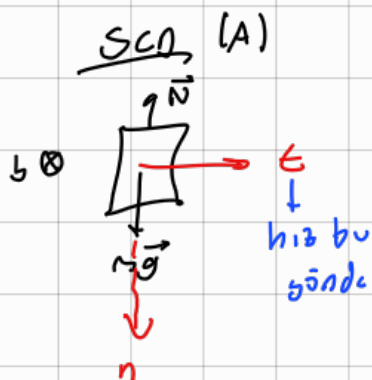
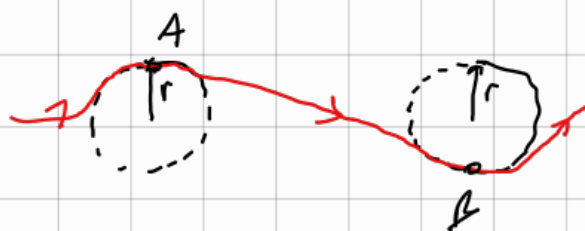
$$= F_n = \frac{4.938m}{\sin \theta} \quad (2)$$

$$(1) \cdot (2) \rightarrow \cot \theta = \frac{mg}{4.938m} \rightarrow \cot \theta = 1.987$$

$$\boxed{\theta = 26.7^\circ}$$

13-75. Determine the maximum speed at which the car with mass m can pass over the top point A of the vertical curved road and still maintain contact with the road. If the car maintains this speed, what is the normal reaction the road exerts on the car when it passes the lowest point B on the road?

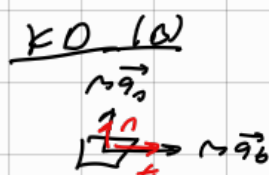
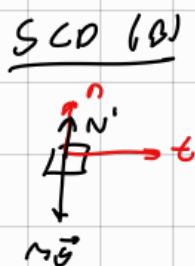




$$\sum F_n = mg - N = m a_n = \frac{mv^2}{R} \rightarrow N = m(g - \frac{v^2}{R})$$

$$\rightarrow v = \sqrt{r(g - \frac{N}{m})}$$

$$N=0 \rightarrow \boxed{v_{\text{ner}} = \sqrt{gr}}$$



$$\sum F_n = N' - mg = \frac{mv^2}{R}$$

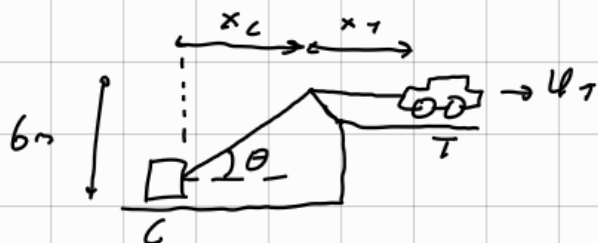
$$v = v_{\text{ner}}$$

$$\boxed{N' = 2mg}$$

WEEK 3, LES 2

Ex 1

If a truck travels at a const. speed of 1.8 m/s, determine the speed of the crate for any angle θ of the rope. The rope has a length of 30m.



$$x_T + \frac{x_C}{\cos \theta} = \text{const} \quad \frac{d}{dt} \rightarrow \underbrace{\dot{x}_T}_{v_T} + \frac{\underbrace{\dot{x}_C}_{v_C} \cos \theta + x_C \sin \theta \dot{\theta}}{\cos^2 \theta} = 0$$

$$v_T + \frac{v_C}{\cos \theta} + \frac{x_C \cdot \sin \theta \cdot \dot{\theta}}{\cos^2 \theta} = 0 \quad (1)$$

$$\tan \theta = \frac{h}{x_C} \quad \frac{d}{dt} \rightarrow \frac{\dot{\theta}}{\cos^2 \theta} = \frac{-h \dot{\theta}}{x_C^2} \quad (2)$$

$$(2) \rightarrow (1): v_T + \frac{v_C}{\cos \theta} - \cancel{x_C} \cdot \sin \theta \cdot \frac{h v_C}{\cancel{x_C^2}} = 0$$

$$(h = \tan \theta \cdot x_C)$$

$$v_T + \frac{v_C}{\cos \theta} - \sin \theta v_C \tan \theta = 0$$

$$v_T + v_C \left(\frac{1 - \sin^2 \theta}{\cos \theta} \right) = 0$$

$$v_T + v_C \cos \theta = 0 \rightarrow v_C = \frac{-v_T}{\cos \theta}$$

$\frac{d\vec{p}}{dt} = \vec{F}$ *moment (m·v)*

$\frac{d(m\vec{v})}{dt} = \vec{F} \rightarrow m \cdot \vec{a} = \vec{F}$
m sabit olursa $F=ma$ 'ye gelir:

$\vec{F} = m\vec{a}$

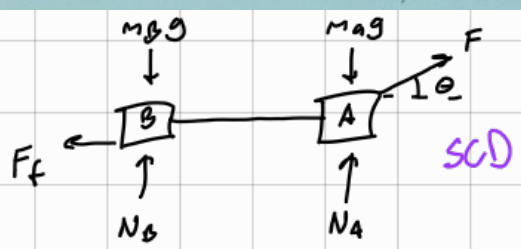
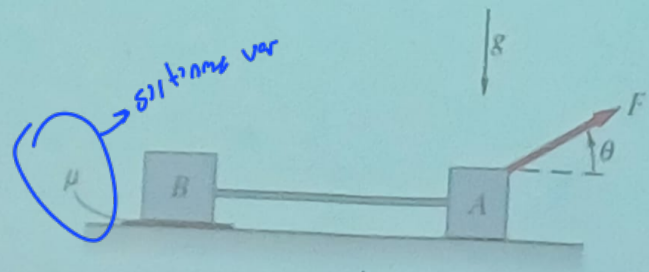
- $m a_x = F_x$
 $m a_y = F_y$ *(Kartezya)*
- $m a_r = F_r$
 $m a_\theta = F_\theta$ *(Kutupsal)*
- $m a_t = F_t$
 $m a_n = F_n$ *(n-t)*

3 ayrı bileşene de geçebilir.

E=P?

Particle kinetics - I

Block A ($m_A = 3 \text{ kg}$) is pulled along a frictionless horizontal surface by a force $F = 50 \text{ N}$ applied at an angle $\theta = 30^\circ$ with respect to the ground. Block B ($m_B = 5 \text{ kg}$) is attached to A by an inextensible cable, and the block slides on top of a rubber mat, nailed onto the surface, with a coefficient of friction $\mu = 0.6$. Find the system's acceleration and the tension in the cable.

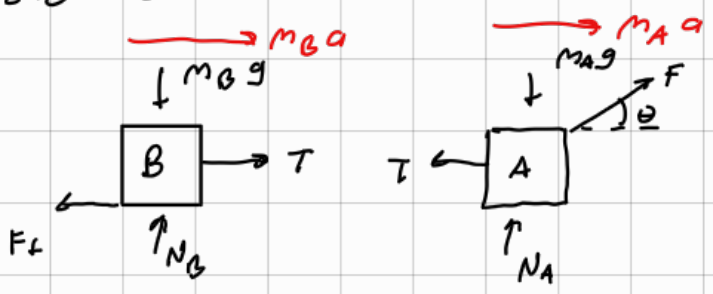


$\sum \vec{F} = m \cdot \vec{a}$

$\sum F_x = m \cdot a_x \Rightarrow F \cos \theta - F_f = (m_A + m_B) \cdot a$ *ipta dales iunle asu*

$F_f = N_B \cdot \mu$

$\sum F_y = 0 \rightarrow N_A + N_B - (m_A + m_B)g + F \sin \theta = 0$



Gözlem sonucu için ayrı ayrı SCD çizildi

B:

$$\sum F_y = 0 \rightarrow m_B g = N_B \rightarrow F_f = m_B g \mu$$

$$\sum F_x = 0 \rightarrow T - F_f = m \cdot a_B$$

$$T - m_B g \mu = m \cdot a_B \quad (1)$$

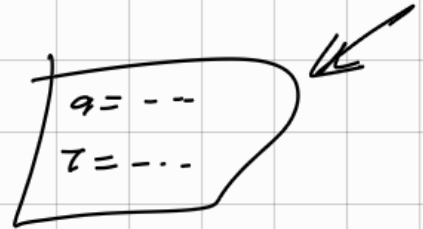
$$(1) \rightarrow T = m_B (a - g \mu)$$

A:

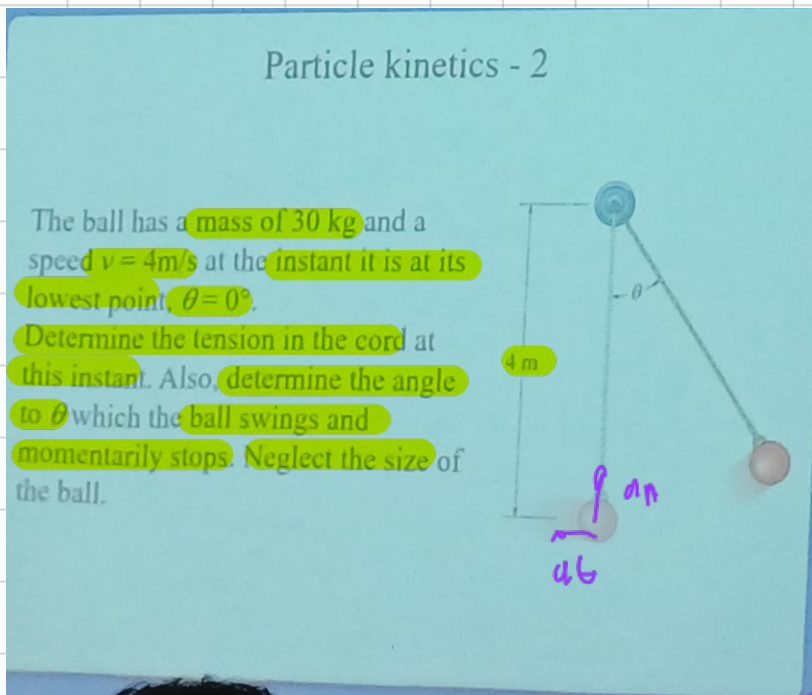
$$\sum F_y = 0 \rightarrow N_A = m_A g - F \sin \theta$$

$$\sum F_x = 0 \rightarrow F \cos \theta - T = m_A a \quad (2)$$

$$(2) \rightarrow F \cos \theta - m_B g \mu = (m_A + m_B) a$$



Ex 2

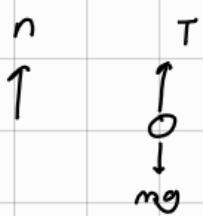


$$a = \dot{\omega} \hat{e}_t + \frac{\omega^2}{r} \hat{e}_n$$

Dağıtılabilir
hacimdeki
 $F = m a$

$$\sum F_n = m \cdot \frac{v^2}{r}$$

1



$$\sum F_n = m \cdot a_n \quad \frac{v^2}{r}$$

$$= m \frac{v^2}{r}$$

$$\rightarrow T - mg = m \frac{v^2}{r}$$

$$\boxed{T = m \left(\frac{v^2}{r} + g \right)}$$

2



$$v = \frac{ds}{dt} = \frac{ds}{d\theta} \cdot \frac{d\theta}{dt} = \frac{v_\theta}{\omega} \rightarrow \int_0^v v dv = \int_0^\theta v_\theta d\theta$$

$$0 = v^2 + 2 \int_0^\theta v_\theta d\theta$$

$$ds = r \cdot d\theta$$

$$+ 2 \cdot \int_0^\theta v_\theta r d\theta$$

v_θ , θ 'ye bağlı olarak değişir

$$v^2 = v_0^2 + 2 \int_0^\theta v_\theta ds$$

dağıtılabilir hacimdeki
tutulu cisimlerin
ve bunların
bütünü.



$$m \cdot a_n \quad m \cdot a_t$$

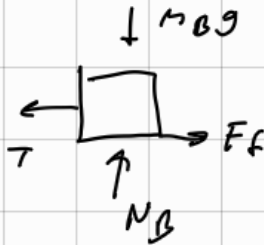
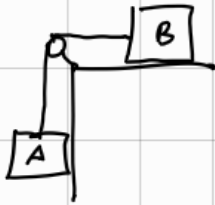
$$\sum F_t = m \cdot a_t$$

$$-mg \sin \theta = m \cdot a_t \rightarrow \underline{a_t = -g \sin \theta}$$

$$0 = v^2 - 2gr \int_0^\theta \sin \theta d\theta \rightarrow \boxed{\cos \theta = \frac{v^2}{2gr}}$$

Ex 3

Determine the acceleration of the blocks when the system is released. The coefficient of kinetic friction is μ_k . Mass of each block is m . Neglect the mass of pulley.



$$(1) \quad T - m_B g \mu = m_B \cdot a$$

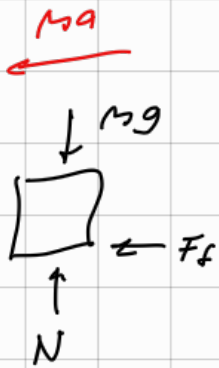
$$(2) \quad m_A g - T = m_A \cdot a$$

(1) $\rightarrow T = \dots$
 (2) $\rightarrow \boxed{a = \dots}$

Exp 4

Coefficient of static friction between the 200kg crate and the flat bed of the truck is $\mu = 0.3$.

Determine the shortest time for the truck to reach a speed of 60 km/h, starting from rest with constant acceleration, so that the crate does not slip.



$$F_f = M \cdot a$$

$$F_f = N \cdot \mu = Mg \mu$$

$$Mg \mu = M \cdot a \rightarrow \boxed{a = g \mu}$$

↓
dimensionless

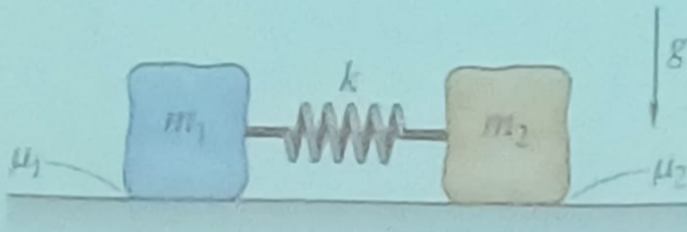
~ WEEK 4 ~

Particle kinetics - 5

Two masses, composed of different materials and having different friction coefficients, are connected by a linear spring with $k = 4 \text{ N/mm}$.

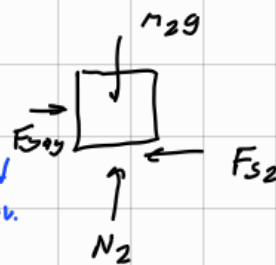
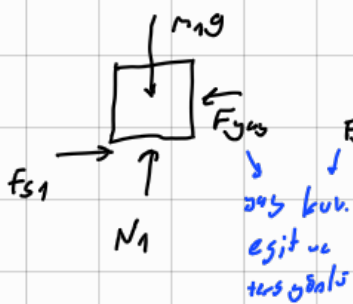
What are the accelerations of the masses when the 5 mm compressed spring is released?

$m_1 = 10 \text{ kg}$, $m_2 = 5 \text{ kg}$, $\mu_1 = 0.4$, $\mu_2 = 0.1$



SCD

$$F_s = \mu \cdot N$$



$$N_1 = m_1 g$$

$$F_{s1} = m_1 g \mu_1$$

$$k \cdot \Delta s - m_1 g \mu_1 = m_1 a_1$$

$$a_1 = \frac{k \Delta s}{m_1} - g \mu_1$$

$$N_2 = m_2 g$$

$$F_{s2} = m_2 g \mu_2$$

$$k \Delta s - m_2 g \mu_2 = m_2 a_2$$

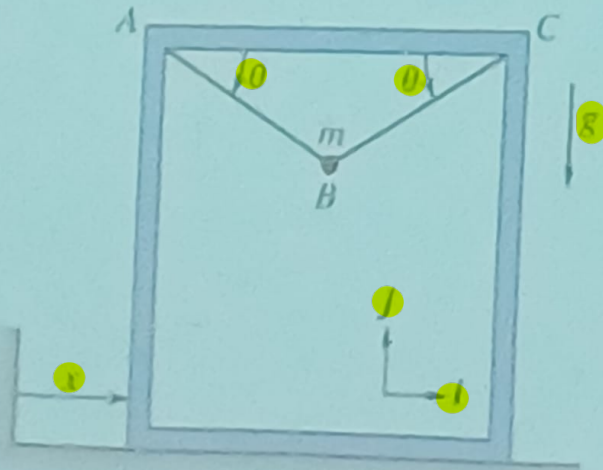
$$a_2 = \frac{k \Delta s}{m_2} - g \mu_2$$

$$F_{spring} = k \cdot \Delta s$$

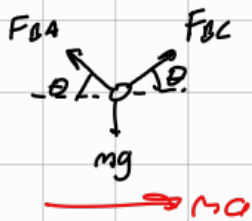
$$F_{spring} = 4 \frac{\text{N}}{\text{mm}} \cdot 5 \text{ mm} = 20 \text{ N}$$

Particle kinetics - 6

A particle m is suspended by two massless, inextensible strings. If the container is moving with the acceleration a , what are the developed forces in the supporting strings?



SCD



$$x: -F_{BA} \cdot \cos \theta + F_{BC} \cdot \cos \theta = ma$$

$$(F_{BC} - F_{BA}) \cdot \cos \theta = ma \rightarrow F_{BC} - F_{BA} = \frac{ma}{\cos \theta} \quad (1)$$

$$y: F_{BC} + F_{BA} \cdot \sin \theta = mg \rightarrow F_{BC} + F_{BA} = \frac{mg}{\sin \theta} \quad (2)$$

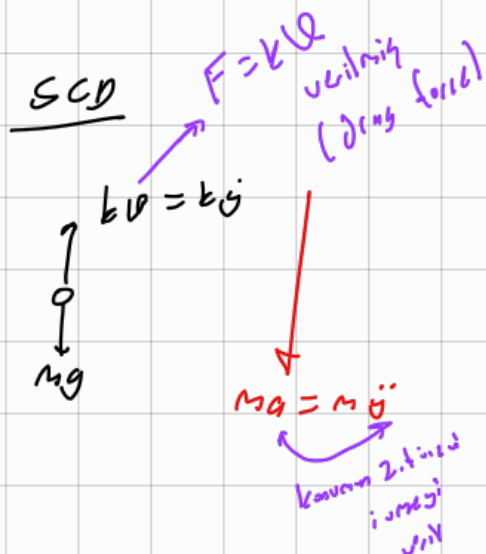
01 .. 01 $\rightarrow$

$$F_{BC} = \frac{m}{2} \left(\frac{a}{\cos \theta} + \frac{g}{\sin \theta} \right) \rightarrow \text{first tends to pull}$$

$$F_{BA} = \frac{m}{2} \left(\frac{g}{\sin \theta} - \frac{a}{\cos \theta} \right) \rightarrow \text{01'den (1) silanacak}$$

Particle kinetics - 7

The rock is dropped from rest within the viscous liquid. If the drag force acting on it is $F = kv$, where k is a constant, determine the velocity and position of the rock as a function of time.



$$\sum F_y = ma$$

$$m_j - k_j = m_j''$$

$$m \frac{d^2 y}{dt^2} + k \frac{dy}{dt} = mg$$

$$m \frac{dv}{dt} + kv = mg$$

Gesamte Kollisionsenergie
 also 1. derse den
 davor sendet:

$$\approx \frac{du}{dt} \quad \text{with } \underline{u=0} \rightarrow \text{horizontal section}$$

$$\frac{du}{dt} = \frac{-1}{2}$$

$$\frac{du}{dt} = -\frac{k}{\rho} dt$$

$$U_h(t) = c_1 \cdot e^{-\frac{t}{\tau}}$$

→ linear 1. durch Subj. ist

$$U_h = C \cdot e^{\delta t}$$

C var stb.

$$C_p = \frac{g_m}{\gamma} \rightarrow 820 / 5200$$

2.2.1.5.2.2.2: $\swarrow$

$m \frac{d\varphi_p}{dt} + k\varphi_p = m\dot{s}$

$\varphi_p = \frac{\dot{s}}{\omega}$

$m \cdot 0 + m\dot{s} = m\dot{s} \checkmark$

$$\varphi(t) = a \cdot e^{\frac{-k}{m}t} + \frac{m}{k}g$$

$$U(0) = 0 \rightarrow c_1 + \frac{1}{k} g = 0$$

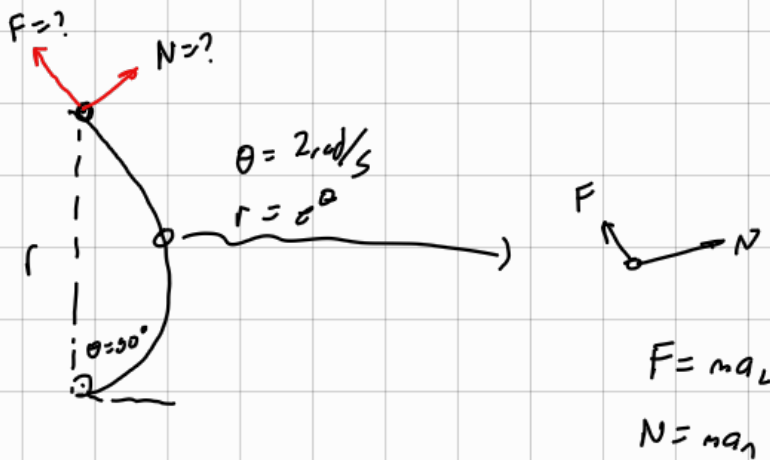
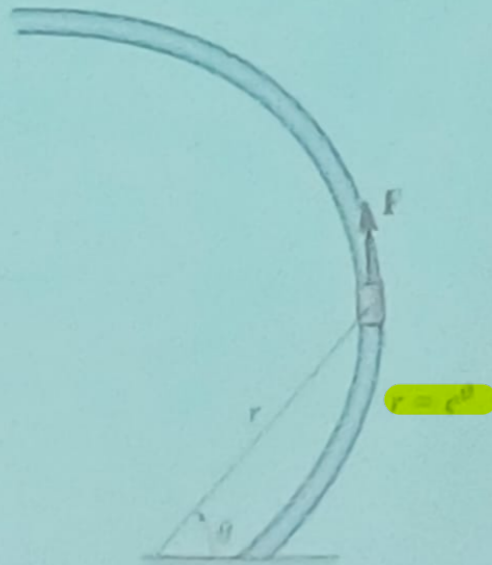
$$u(t) = \frac{\gamma}{b} g \left(1 - e^{-\frac{b}{\gamma} t} \right)$$

$$S = \int u \, dx$$

$\gamma = \dots$

Particle kinetics - 8

The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 90^\circ$ if the force F maintains a constant angular velocity of 2 rad/s.



$$\vec{a} = (\ddot{r} + r\dot{\theta}^2) \vec{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta}) \vec{e}_\theta = \dot{v} \vec{e}_t + \frac{v^2}{\rho} \vec{e}_n$$

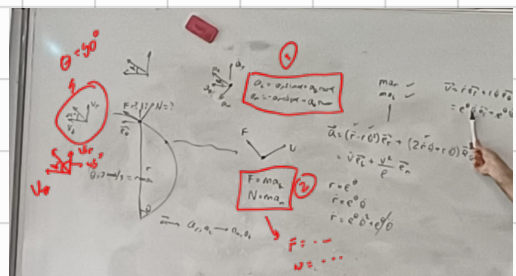
$$r = e^\theta$$

$$\dot{r} = e^\theta \cdot \dot{\theta}$$

$$\ddot{r} = e^\theta \cdot \dot{\theta}^2 + e^\theta \cdot \ddot{\theta}$$

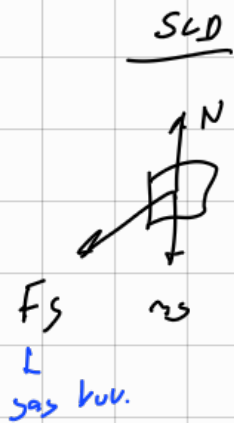
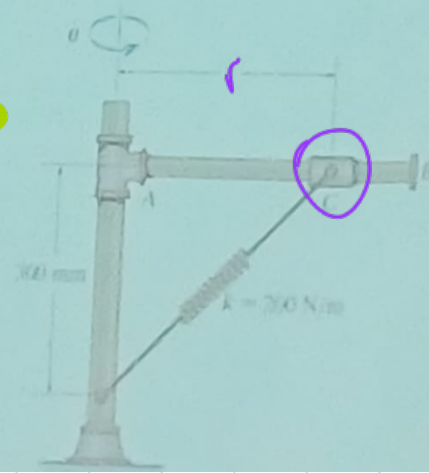
$$\vec{v} = \dot{r} \cdot \vec{e}_r + r \dot{\theta} \cdot \vec{e}_\theta = e^\theta \cdot \dot{\theta} \vec{e}_r + e^\theta \cdot \dot{\theta} \vec{e}_\theta$$

Derivati →



Particle kinetics - 9

If the position of the 3-kg collar C on the smooth rod AB is held at $r = 720 \text{ mm}$, determine the constant angular velocity ω which the mechanism is rotating about the vertical axis. The spring has an unstretched length of 400 mm. Neglect the mass of the rod and the size of the collar.



$$\sum \vec{F}_n = m \cdot \vec{a}_n \rightarrow F_s \cdot \sin \alpha = m \cdot a_n$$

$$F_s = k \cdot \Delta s$$

$$\Delta s = \sqrt{0.72^2 + 0.2^2} - 0.4$$

$$m \cdot a_n = m \cdot \frac{v^2}{r}$$

$$= m \cdot \dot{\theta}^2 r$$

$$\left(\vec{a} = \cancel{\dot{r}} \vec{e}_r + \frac{v^2}{r} \vec{e}_n \right)$$

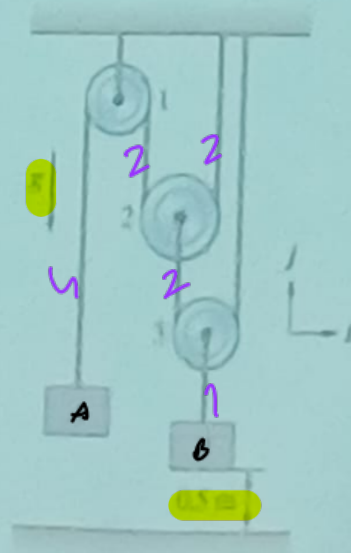
$$\omega = \dot{\theta} = \dot{\varphi} \cdot r$$

↓ ↓
equal to

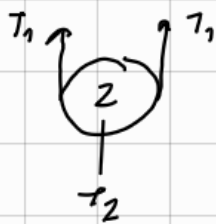
$$\dot{\theta} = \sqrt{\frac{F_s \cdot \sin \alpha}{m r}}$$

Particle kinetics - 11

The pulley system is released from rest. How much time is needed for B to contact the ground? $m_A = 20 \text{ kg}$, $m_B = 160 \text{ kg}$. The pulleys and ropes are massless.



$$T_1 - m_A g = m_A a_A \quad (1)$$

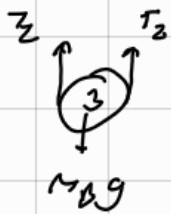


Wicht vor sich

habe abwärts

$$2T_1 = T_2$$

Ges. steht



$$m_B g - 4T_1 = m_B a_B \quad (2)$$



$$(1), (2) \text{ u. } (3) \rightarrow a_B = \dots$$

$$a_A = \dots$$

$$a_A = 4a_B \quad (3)$$

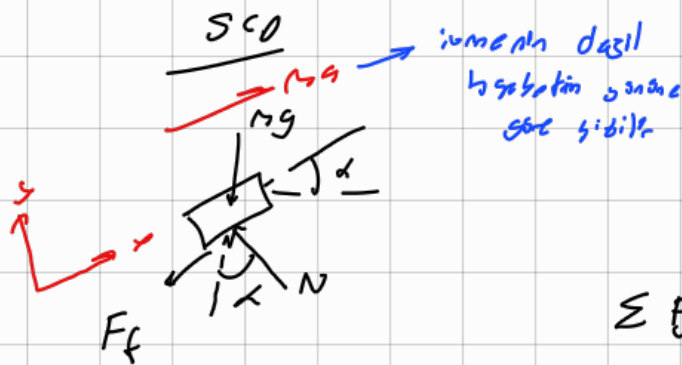
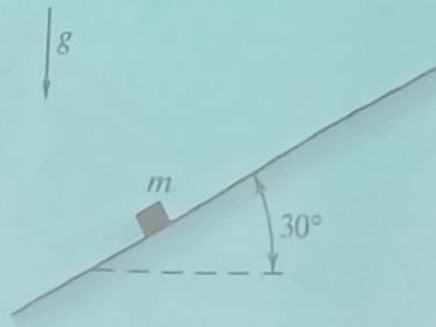
→ relative scheinbar
schieben d.h. 1



Particle kinetics - 12

A 2-kg mass m is projected up a 30° slope with an initial speed of 10.2 m/s . The mass comes to rest at a point that's 5 m higher in elevation than its starting position.

What must be the coefficient of dynamic friction between the mass and the slope?



$$\Sigma F = m a$$

$$F_f = N \mu$$

$$\Sigma F_y = 0 \rightarrow N = m g \cos \alpha$$

$$\Sigma F_x = m a \rightarrow -m g \sin \alpha - \mu m g \cos \alpha = m a$$

$$a = -g (\sin \alpha + \mu \cos \alpha)$$

$$v_1^2 = v_0^2 + 2 a x$$

0

L

$$2g(\sin \alpha + \mu \cos \alpha) x = v_0^2 \rightarrow \mu = \dots$$

$\downarrow$
 $h/\cos \alpha$

Her würde besser separat sein
das ist seine senkrechte Wurfhöhe
Er soll das hier vorne geben.

~ İş - Enerji ilişkisi ~

$$\frac{d}{dt}(m\vec{v}) = \vec{F} \rightarrow \vec{v} \cdot \frac{d\vec{r}}{dt}$$

$$m \frac{d\vec{v}}{dt} \cdot d\vec{r} = \vec{F} \cdot d\vec{r}$$

$$\cdot \vec{v} dt$$

$$\int_{v_1}^{v_2} m d\vec{v} \cdot \vec{v} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

$$\int_{v_1}^{v_2} d\left(\frac{1}{2} m \vec{v} \cdot \vec{v}\right) = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} \rightarrow \left(\frac{1}{2} m v_2^2\right) - \frac{1}{2} m v_1^2 = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} \rightarrow (T_2 - T_1 = W_{12})$$

kinetik enerji

*bi. birim
Joule*

~ İTME - MOMENTUM ~

$$\frac{d}{dt}(m\vec{v}) = \vec{F}$$

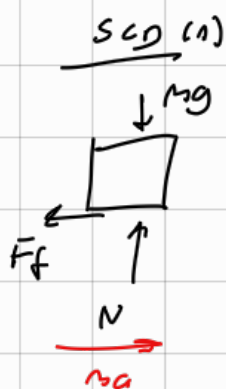
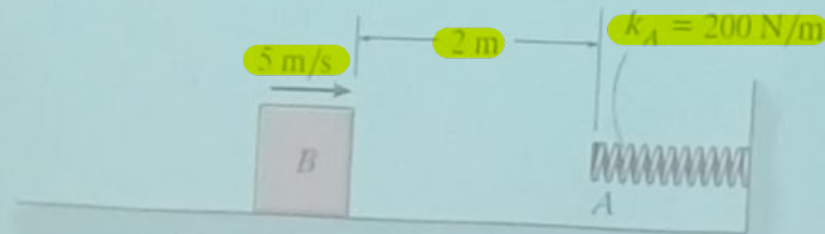
$$\int_{t_1}^{t_2} d(m\vec{v}) = \int_{t_1}^{t_2} \vec{F} dt$$

$$\vec{p}_2 - \vec{p}_1 = \int_{t_1}^{t_2} \vec{F} dt$$

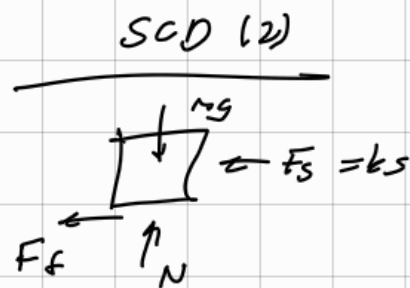
İTME

Particle kinetics - 13

The 8-kg block is moving with an initial speed of 5 m/s. If the coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$, determine the compression in the spring when the block momentarily stops.



$$\begin{aligned} \downarrow: N &= mg \\ \leftarrow: F_f &= ma \\ \mu mg &= ma \rightarrow \mu g = a \end{aligned}$$



$$T_1 + W_{1,2} = T_2$$

$$\frac{1}{2}mv^2 - \mu mg(2+s) - \int_0^s k \cdot s \, ds = 0$$

$\downarrow$
 $-\frac{1}{2}ks^2 \rightarrow$ *spring pot. energy?*

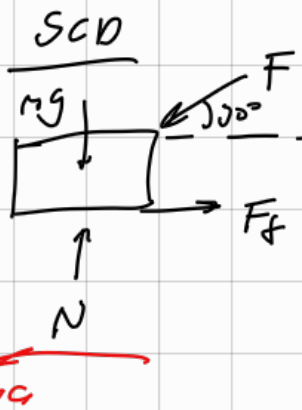
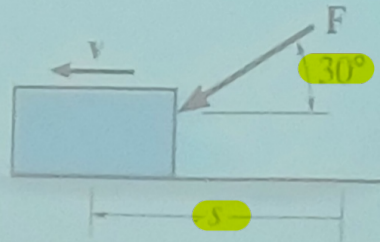
$$\frac{1}{2}mv^2 - \mu mg(2+s) - \frac{1}{2}ks^2 = 0 \rightarrow \boxed{s = \dots}$$

Particle kinetics - 14

buvarit s'ic bagli, interst friends bullethead!!

The 2-kg block is subjected to a force having a constant direction and a magnitude $F = 300/(1+s)$, where s is in meters. When $s = 4$ m, the block is moving to the left with a speed of 8 m/s.

Determine its speed when $s = 12$ m. The coefficient of kinetic friction between the block and the ground is $\mu_k = 0.25$.



$$T_1 + W_{12} = T_2$$

$$W_{12} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

$$\frac{1}{2} m v_1^2 + \int_4^{12} \frac{300}{1+s} \cos 30^\circ ds - \mu \int_4^{12} \left(mg + \frac{300}{1+s} \sin 30^\circ \right) ds = \frac{1}{2} m v_2^2$$

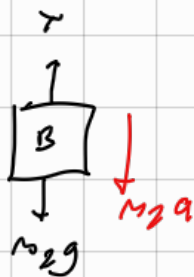
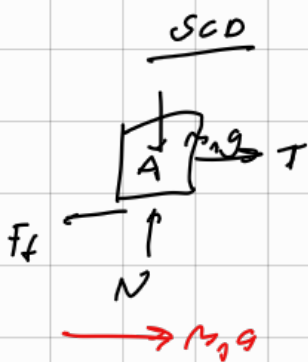
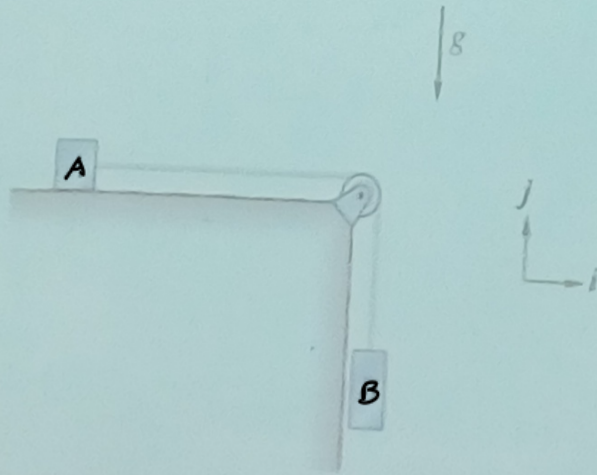
$$\sum F_y = 0 : N = F \sin 30^\circ + mg$$

$$F_f = \mu \left(mg + \frac{300}{1+s} \sin 30^\circ \right) = \frac{1}{2} m v_1^2 - \mu mg (12-4) + 300 \cdot (\cos 30^\circ - \sin 30^\circ) \cdot \underbrace{\int_4^{12} \frac{ds}{1+s}}_{\ln(1+s) \Big|_4^{12}} = \frac{1}{2} m v_2^2$$

$$v_2 = \dots$$

Particle kinetics - 15

A massless rope runs from block A, over a massless, frictionless pulley and attaches to block B. The coefficient of friction between the block A and the table are $\mu_s = \mu_d = 0.6$ and $m_A = 10 \text{ kg}$, $m_B = 20 \text{ kg}$. Assuming that the system is released from rest, what will block B's speed be after it has moved 0.5 m?



system at rest

$$\cancel{T_1} + W_{1 \rightarrow 2} = T_2$$

↓
kinetic energy

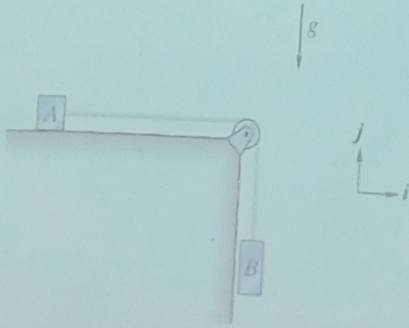
$$-\underbrace{\mu m g \cdot s}_{0.6m} + m_B g s + \underbrace{\cancel{T \cdot 0.5} - \cancel{T s}}_{\text{kinetic energy}} = \frac{1}{2} m v^2 \rightarrow v_2 = \dots$$

~ WEEK 5 ~

Particle kinetics - 15

→ full system
Energy known
unknown!!

A massless rope runs from block A, over a massless, frictionless pulley and attaches to block B. The coefficient of friction between the block A and the table are $\mu_s = \mu_d = 0.6$ and $m_A = 10 \text{ kg}$, $m_B = 20 \text{ kg}$. Assuming that the system is released from rest, what will block B's speed be after it has moved 0.5 m?



is, Energy, in the RENELEMUSI

$$\sum \vec{F} = m \vec{a}$$

$$T_1 + W_{1-2} = T_2$$

$$(\vec{r} \times \vec{L})_1 + \int_{t_1}^{t_2} \vec{r} \times \vec{F} dt = (\vec{r} \times \vec{L})_2$$

$$W_{1-2} = U_1 - U_2 = -\Delta U$$

$$U_{\text{sp}} = \frac{1}{2} k x^2$$

$$T_1 + U_1 + W_{1-2} = T_2 + U_2$$

$$(\vec{r} \times m \vec{v})_1 + \int_{t_1}^{t_2} (\vec{r} \times \vec{F}) dt = (\vec{r} \times m \vec{v})_2$$

$$T_1 + U_1 + W_{1-2} = T_2 + U_2$$

$$0 + 0 + \underbrace{\mu m_A g s}_{F_k = N \mu} = \frac{1}{2} (m_A + m_B) v^2 - m_B g s$$

$$v^2 = \frac{(m_B - \mu m_A) \cdot g s}{m_A + m_B}$$

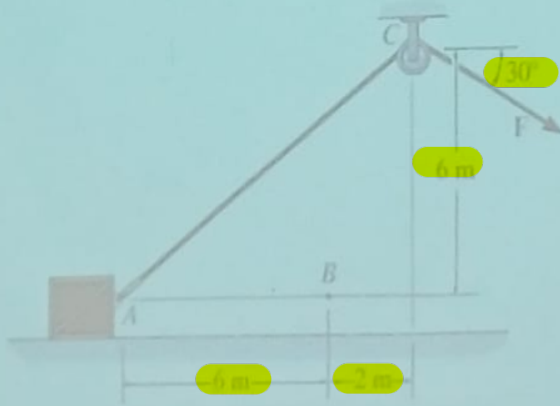
$$\sum F = m a \rightarrow a = \frac{m_B g - \mu m_A g}{m_A + m_B}$$

$$v^2 = 2 a s \rightarrow v = \sqrt{2 a s}$$

$$\frac{2 (m_B - \mu m_A) g}{m_A + m_B} \cdot s$$

Particle kinetics - 16

If the 75-kg crate starts from rest at A, determine its speed when it reaches point B. The cable is subjected to a constant force of $F = 300$ N. Neglect friction and the size of the pulley.

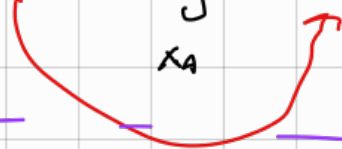


→ Since better solve

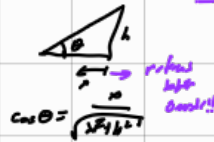


$$T_A + W_{AO} = T_B$$

$$\int_{x_A}^{x_B} F \cdot \cos \theta(x) dx = \frac{1}{2} m v_B^2$$



$$F \cdot \int_0^2 \frac{x}{\sqrt{x^2 + h^2}} dx = \frac{1}{2} m v_B^2$$



$$x^2 + h^2 = u \rightarrow 2x dx = du$$

$$\left. -\frac{F}{2} \int_{u_1}^{u_2} u^{-1/2} du = -F \sqrt{x^2 + h^2} \right|_0^2 = \frac{1}{2} m v_B^2$$

$U = \dots$

$$\left. -\frac{F}{2} \frac{u^{1/2}}{1/2} \right|_{u_1}^{u_2}$$

VEYA

$$\cos \theta = \frac{8-x}{\sqrt{(8-x)^2 + h^2}}$$

$$F \int_0^6 \frac{8-x}{\sqrt{(8-x)^2 + h^2}} dx = \frac{1}{2} m v_B^2$$

$$(8-x)^2 + h^2 = u \rightarrow -2(8-x) dx = du$$



$$\left. -\frac{F}{2} \frac{u^{1/2}}{1/2} \right| \rightarrow -F \sqrt{(8-x)^2 + h^2} \Big|_0^6 = \frac{1}{2} m v_B^2 \rightarrow U = \dots$$

Particle kinetics - 18

The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring, which is attached to the cart and not the block, is compressed 0.2 m and the system is released from rest, determine the speed of the block with respect to the cart after the spring becomes undeformed. Neglect the mass of the wheels and the spring in the calculation and also neglect friction. Take $k = 300 \text{ N/m}$.

~~$$T_1 + U_1 = T_2 + U_2$$~~

$$\frac{1}{2} k \Delta s^2 = \frac{1}{2} m_B v_B^2 + \frac{1}{2} m_C v_C^2 \quad (1)$$

↓
Yes pot.

$$m v_1 + \int F dt = m v_2$$

disada bu. akhir

$$0 = m_B v_B - m_C v_C \quad (2)$$

✓
 $v_1 = 0$

$$(1) \text{ \& } (2) \rightarrow v_B = \sim, v_C = \sim \left\{ \begin{array}{l} v_B = v_C + v_{B/C} \\ v_{B/C} = v_B - v_C \end{array} \right.$$

bas. l h. d. akhir

N WEEK 5, LESSON 2 ~

$\vec{r} \times m \vec{v} \rightarrow$ angular momentum

$$\vec{r} \times \sum \vec{F} = \frac{d}{dt} (\vec{r} \times m \vec{v})$$

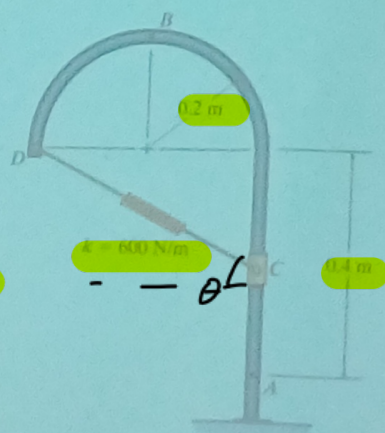
$$\int_{t_1}^{t_2} dt (\vec{r} \times \sum \vec{F}) = \int d(\vec{r} \times m \vec{v})$$

$$\int_{t_1}^{t_2} (\vec{r} \times \sum \vec{F}) dt = (\vec{r} \times m \vec{v})_2 - (\vec{r} \times m \vec{v})_1$$

$$(\vec{r} \times m \vec{v})_1 + \int_{t_1}^{t_2} (\vec{r} \times \sum \vec{F}) dt = (\vec{r} \times m \vec{v})_2$$

Particle kinetics - 18

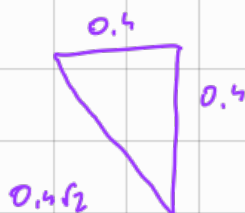
The 2-kg collar is released from rest at A and travels along the smooth vertical guide. Determine the speed of the collar when it reaches position B. Also find the normal force exerted on the collar at this position. The spring has an unstretched length of 200 mm.



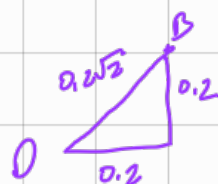
$$T_1 + U_1 = T_2 + U_2$$

$$\cancel{T_1} + \cancel{U_{s1}} + \cancel{U_{g1}} = T_2 + U_{s2} + U_{g2}$$

$$\frac{1}{2} k (0.4\sqrt{2} - 0.2)^2 = \frac{1}{2} m v_B^2 + \frac{1}{2} k (0.2\sqrt{2} - 0.2)^2 + m g \cdot 0.6 \rightarrow v_B = \dots$$



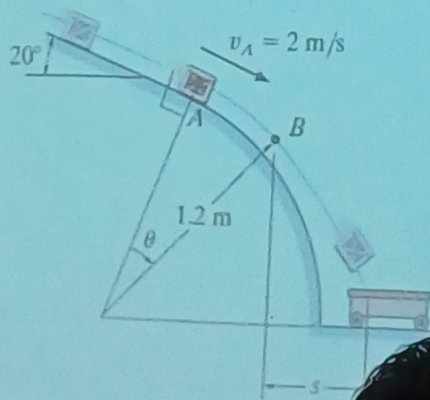
Stretch length = $0.4\sqrt{2} - 0.2$
 ↓
 unstretched length



unstretched length = $0.2\sqrt{2} - 0.2$
 unstretched length

Particle kinetics - 19

When the 6-kg box reaches point A it has a speed of $v_A = 2 \text{ m/s}$. Determine the angle θ at which it leaves the smooth circular ramp and the distance s to where it falls into the cart. Neglect friction.



$N = 0$ oldas en?



$$\sum F_A = m \cdot a_n$$

$$mg \sin \alpha - N = m a_n = m \frac{v^2}{r}$$

$$\left(a_n = \frac{v^2}{r} \right)$$

$$T_A + U_A = T_B + U_B$$

$$\frac{1}{2} m v_A^2 + m g (h_A - h_B) = \frac{1}{2} m v_B^2$$

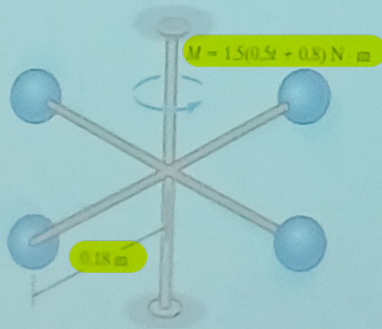
$$v_B^2 = v_A^2 + 2 \int_A^B a_t ds$$

$$\theta = \dots$$

$$s = \dots$$

Particle kinetics - 20

The four 2.5-kg spheres are rigidly attached to the crossbar frame having a negligible weight. If a couple moment is applied as shown, determine the angular speed of the crossbar in 4 seconds starting from rest.



Niraxds paraxiloch
naxiloch güt st !!

$$\cancel{T_1} + W_{12} = T_2$$

$$\text{b. } W = \int \vec{F} \cdot d\vec{r} \Rightarrow \int_{\theta_1}^{\theta_2} m \, d\theta$$

$$\int_0^{\theta_x} M(\theta) d\theta$$

$M(f)$ dB

could nonorthogonality is

$t=0$ signal bandwidth

$$\int_{\theta_1}^{\theta_2} M(\theta) d\theta = \int_{t_1}^{t_2} M(t) \dot{\theta}(t) dt$$

$$\int_0^4 M(t) dt = \vec{r} \times (m \vec{v}) \quad \text{L608 durch...}$$

$$= 4 \cdot 0.18 \cdot 2.5 \cdot (0.18 \dot{\theta}) \rightarrow \int$$

11 Gründe
ausser noch
50, haarkitz

akumulasi momentum
(600 durasi)

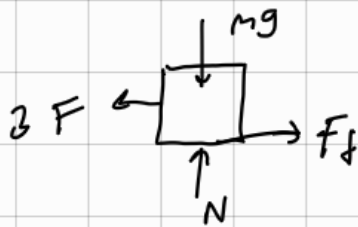
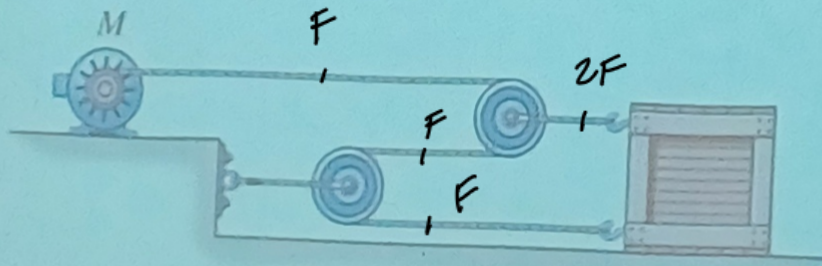
(nicht voll)

$$\varphi = r \cdot \omega = r \cdot \dot{\theta}$$

→ $\dot{\theta} = \dots$

Particle kinetics - 21

The crate has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are 0.3 and 0.2, respectively. If the motor M supplies a cable force of $F = 8t^2 + 20$, where t is in seconds, determine the power output developed by the motor when $t = 5$ s.



$$3F - F_f = ma$$

$$3F - \mu mg = ma \rightarrow a = \frac{1}{150} (3 \cdot (8t^2 + 20) - 0.3 \cdot 150g)$$

$$a = \frac{8t^2 + 20}{50} - 0.3g$$

$$U_B = \int_a^b a \, dt$$

Sartlamada usulda buvurda
bosh olmas, iqtin:
(hazirki boshlamay iqtin)

$$3F = \mu mg \rightarrow 3(8t^2 + 20) = 0.3 \cdot 150 \cdot g \rightarrow t_0$$

$$U_B = \int_{t_0}^5 a \, dt \rightarrow U_B = \dots$$

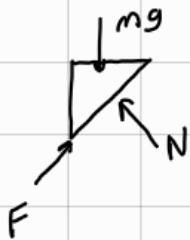
$$U_M = 2 \cdot U_B \rightarrow U_M = \dots$$



~ WEEK 6 ~

$\vec{u}'$

$u_{mi} \cdot \cos \theta$



Özden derhalte
kurat atırıldığında
momentum korunur!!

seri f. yoğ

$\downarrow$ hız edildi ??

$$m \cdot u_i \cdot \cos \theta = (m + M) \cdot u' \quad \text{← hızı en azdır}$$

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) \cdot u_3$$

$$\frac{1}{2} (m + M) u'^2 = (m + M) g h \rightarrow \boxed{h = \dots}$$

$$u_1 + v_1 = u_2 + v_2$$

relative
determine
as a
mass of

A-B sistemi: $\swarrow$ 2'ye bishar

$$0 = m_A v_A + m_B v_B = m_A v_A + m_B (v_A - v_{0/A} \cdot \cos \theta) \Rightarrow v_A (m_A + m_B) = -m_B \cdot v_{0/A} \cos \theta$$

$$v_A = -v_{0/A} \cdot \frac{m_B}{m_A + m_B} \rightarrow \boxed{v_A = -}$$

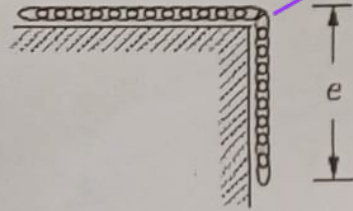
B-C sistemi:

$$(m_C + m_B) \cdot 0 + m_B v_B = (m_B + m_C + m_D) \cdot v_C \rightarrow \boxed{v_C = \dots}$$

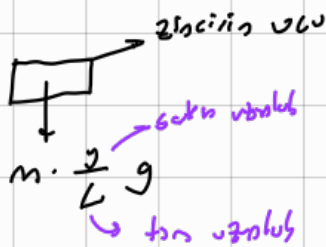
$$v_B = v_A + v_{0/A} \cdot \cos \theta$$

Particle kinetics - 25

A chain with length l and mass m hangs over the edge of a frictionless table by an amount e . If the chain starts with zero initial velocity, find the position of the end of the chain as a function of time.



Kette ve
uygulan kuvvet
sabit olduğu için!



$$ma = m\ddot{y}$$

$$\frac{m y}{L} \cdot g = m \ddot{y} \rightarrow \frac{d^2 y}{dt^2} - \frac{g}{L} y = 0$$

$$y = e^{rt} \rightarrow r^2 - \frac{g}{L} = 0 \rightarrow r_{1,2} = \pm \sqrt{\frac{g}{L}}$$

homogen eşitlik

$$y(t) = c_1 \cdot e^{\sqrt{\frac{g}{L}} t} + c_2 \cdot e^{-\sqrt{\frac{g}{L}} t}$$

VEZA:

$$= A \cosh \sqrt{\frac{g}{L}} t + B \cdot \sinh \sqrt{\frac{g}{L}} t \rightarrow$$

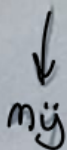
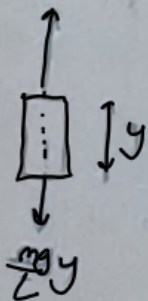
$$v_0 = \dot{y}(0) = 0 \rightarrow \underline{B=0}$$

$$y(0) = e \rightarrow \underline{e=A}$$

hiperboller de kullanılabilir.

Üstteki sarda sarkma olası:

$$\mu m \frac{L-y}{L} g$$



$$m\ddot{y} = \frac{mg}{L} (1-\mu) y - \mu mg$$

$$\frac{d^2 y}{dt^2} - \frac{g}{L} (1-\mu) y = -\mu mg$$

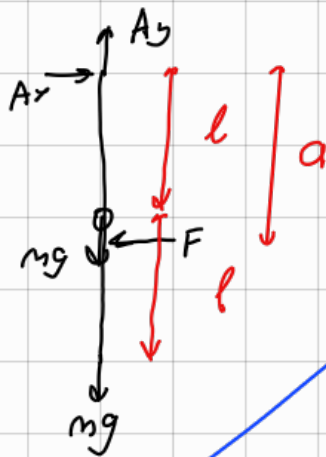
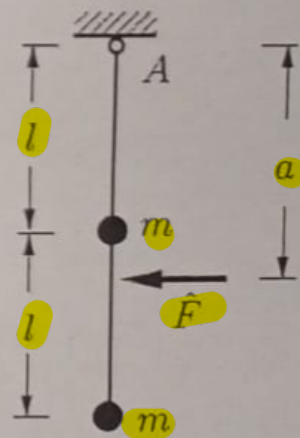
$$y = A e^{-\sqrt{\frac{g}{L}(1-\mu)} t} + B e^{\sqrt{\frac{g}{L}(1-\mu)} t} + \frac{\mu mg}{\frac{g}{L}(1-\mu)}$$

Particle kinetics - 26

The rigid rod carries two point masses.

It is struck by an impulsive force F at a distance a from the support A .

Determine the angular velocity of the rod immediately after the impact and the impulsive reaction at A . Also calculate a so that the reaction force at A is zero.



$$\vec{M} = \frac{d}{dt} \left(\sum_i (\vec{r}_i \times m_i \vec{v}_i) \right)$$

$$\int M dt = d()$$

with sign convention

$$\int (\vec{r} \times \vec{F}) dt \rightarrow \int_0^t a F dt = a \cdot F = m \dot{\theta} l^2 + m \dot{\theta} (2l)^2$$

$\downarrow$ $\downarrow$
 $(\dot{\theta} \cdot l = v_1)$ $(\dot{\theta} \cdot 2l = v_2)$

$$a F = m \dot{\theta} \cdot l^2 (1+4) \rightarrow \boxed{\dot{\theta} = \frac{a \cdot F}{5 m l^2}} \text{ (rad/s)}$$

~WEEK 6, LESSON 2~

Dışarıda kuvvet uygulanan maddelerin momentumları korunur.

$$m_1 u_1 + m_2 u_2 = m_1 u_1' + m_2 u_2' \quad (1)$$

$$e = \frac{\int P dt}{\int P dt}$$

ayrıştırma faktörü
sistemler arasındaki impuls
katsayı, değeri belirlenir

Çarpışan cisimlerin hızları, bir önceki hızlarından farklıdır.

$$2 \text{ çarpışan cisim} \rightarrow e = 0.6 - 0.8$$

$$2 \text{ cisim} \text{ çarpışır} \rightarrow e = 0.95$$

$$e = 0 \rightarrow \text{plastik çarpışma (cisimler birleşir)}$$

$$m_A u_A - \int P dt = m_A u \rightarrow \text{sistemler arasındaki impuls}$$

$$m_A u - \int P dt = m_A u_A' \rightarrow \text{ayrıştırma faktörü}$$

$$e = \frac{u - u_A'}{u_A - u}$$

ayrıştırma faktörü

$$m_B u_B + \int P dt = m_B u$$

$$m_B u + \int P dt = m_B u_B'$$

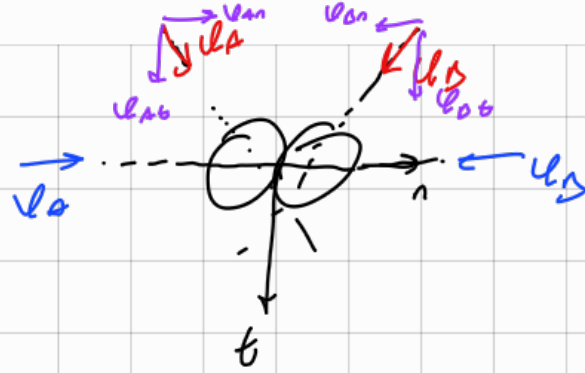
$$e = \frac{u_B' - u}{u - u_B}$$

$$u_A' + e u_A = e u_B + u_B' \rightarrow e(u_A - u_B) = u_B' - u_A'$$

$$e = \frac{u_B' - u_A'}{u_A - u_B} \quad (2)$$

Çarpışan cisimlerin hızları

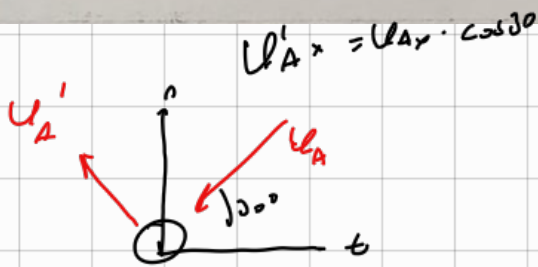
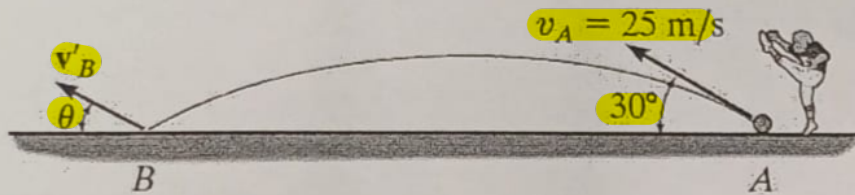
Çarpışan cisimlerin hızları



→ normal hızı (normal)
 → teğet hızı (teğet)

Impact - 1

A 300-g ball is kicked with a velocity of $v_A = 25 \text{ m/s}$ at point A as shown. If the coefficient of restitution between the ball and the field is $e = 0.4$, determine the magnitude and direction θ of the velocity of the rebounding ball at B.



$$e = \frac{u_B' - u_A'}{u_A - u_B} = - \frac{u_{An}'}{u_{An}} \rightarrow u_{An}' = -e u_{An}$$

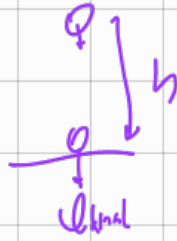
$$m_A u_{An} = m_A \cdot u_{An}' \rightarrow u_{An} = u_{An}'$$

momentum korunur dedi, fakat yanlış
 için yanlış.

E-17

⊙ A

Çarpışmadan hemen sonraki hızlar?



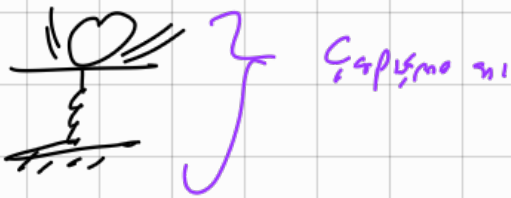
$$m \sqrt{2gh} + 2m \cdot 0 = m v_A' + 2m v_B'$$

$$e = \frac{v_B' - v_A'}{\sqrt{2gh} - 0} \rightarrow v_B' = e \sqrt{2gh} + v_A'$$

$\downarrow$ ideal $\Rightarrow 1$

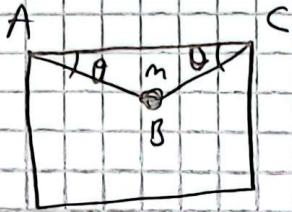
$$v_A' \cdot m (1+2) = m \sqrt{2gh} (1-2e) \rightarrow v_A' = \sqrt{2gh} \cdot \frac{1-2e}{3}$$

Çarpışmadan hemen sonraki hız durumları tabii ki, yazmamız lazım.

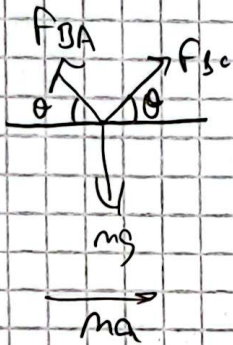


~ WEEK 7

$$v = \omega r$$



FBD



$$F_{BA} \sin \theta + F_{BC} \sin \theta = mg$$

$$-F_{BA} \cos \theta + F_{BC} \cos \theta = ma$$

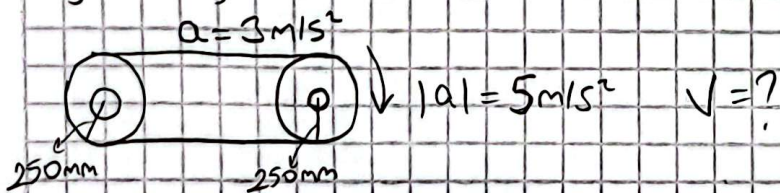
$$F_{BC} - F_{BA} = ma / \cos \theta$$

$$F_{BC} + F_{BA} = mg / \sin \theta$$

$$F_{BC} = \frac{m}{2} \left(\frac{g}{\cos \theta} + \frac{g}{\sin \theta} \right)$$

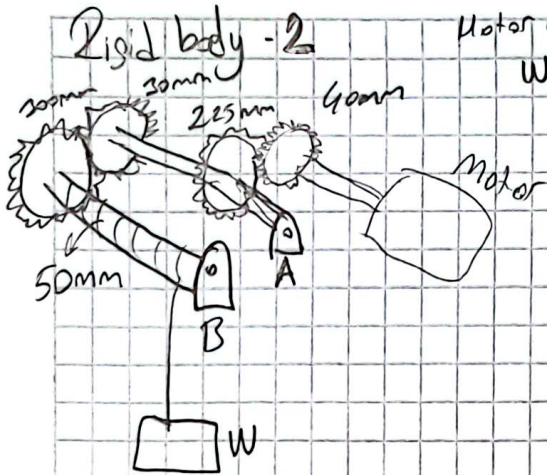
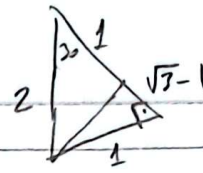
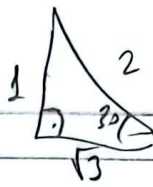
$$F_{BA} = \frac{m}{2} \left(\frac{g}{\sin \theta} - \frac{g}{\cos \theta} \right)$$

Rigid body kinematics. 1



$$5 \text{ m/s}^2 = \sqrt{a_t^2 + a_n^2} = \sqrt{3^2 + \left(\frac{v^2}{r}\right)^2}$$

$$\frac{v^2}{r} = 4$$



Motor angular velocity
 $W = 100(4+t) \text{ rad/s}$

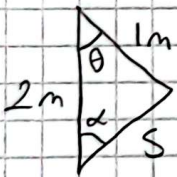
$$W \cdot 40 = W_A \cdot 225$$

$$W_A \cdot 30 = W_B \cdot 300$$

$$W = W_B \cdot 50$$

$$h = \int_0^5 v \cdot dt$$

Rigid body - 4

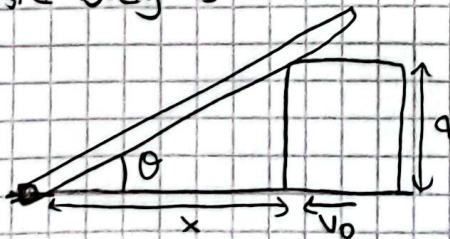


$$S^2 = 1^2 + 2^2 - 2 \cdot 1 \cdot 2 \cdot \cos \theta$$

$$2S \dot{S} = 2 \cdot 1 \cdot 2 \sin \theta \dot{\theta}$$

$$W = \frac{S \dot{S}}{2S \sin \theta}$$

Rigid body - 5



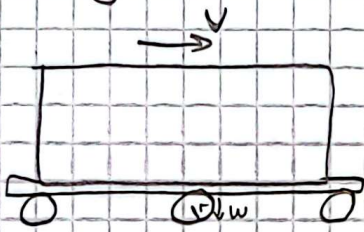
Determine angular velocity and angular acceleration of function of θ the bar

$$\cot \theta = \frac{x}{a} \rightarrow -\frac{\dot{\theta}}{\sin^2 \theta} = \frac{\dot{x}}{a} \rightarrow -\frac{W}{\sin^2 \theta} = \frac{v_0}{a}$$

$$W = \frac{v_0}{a} \sin^2 \theta$$

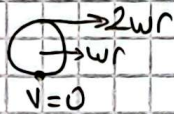
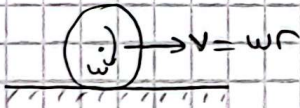
$$\alpha = 2 \frac{v_0}{a} \sin \theta \cos \theta \dot{\theta} + \frac{v_0}{a} \sin^2 \theta$$

Rigid body - 6



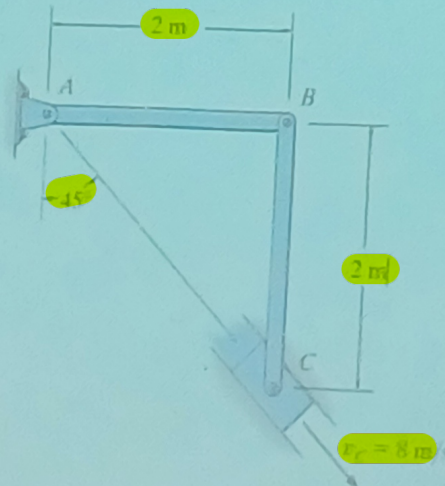
Determine angular velocity with a v .

$$v = 2r\omega \Rightarrow \omega = \frac{v}{2r}$$



~ WEEK 7, LES 2 ~

The slider block C moves at 8 m/s down the inclined groove. Determine the angular velocities of links AB and BC, at the instant shown.



$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$$

$$\omega d\omega = \alpha d\theta$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\begin{aligned} \vec{a} &= \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) \\ &= \vec{\alpha} \times \vec{r} - \omega^2 \vec{r} \end{aligned}$$

$$\left. \begin{aligned} \vec{r}_C &= \vec{r}_B + \vec{r}_{C/B} \\ \vec{v}_C &= \vec{v}_B + \vec{v}_{C/B} \\ \vec{a}_C &= \vec{a}_B + \vec{a}_{C/B} \end{aligned} \right\} \text{by same C relation}$$

$$\omega_{AB} \vec{i} \times r_{AB} \vec{j} = \omega_{AB} \cdot r_{AB} \vec{k}$$

$$\vec{v}_C = \vec{v}_B + \vec{v}_{C/B} = \vec{\omega}_{AB} \times \vec{r}_{AB} + \vec{\omega}_{BC} \times \vec{r}_{BC}$$

$$\omega_{BC} \vec{i} \times r_{BC} \vec{j} = \omega_{BC} \cdot r_{BC} \vec{k}$$

$$\vec{v}_C = v_C (\cos 45^\circ \vec{i} - \sin 45^\circ \vec{j})$$

$$v_C = 2\omega_{AB} \vec{j} + 2\omega_{BC} \vec{i} = v_C (\cos 45^\circ \vec{i} - \sin 45^\circ \vec{j})$$

$$i: 2\omega_{BC} = 8 \cos 45^\circ \quad (1)$$

$$j: 2\omega_{AB} = -8 \sin 45^\circ$$

$$\vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{AB} = \vec{v}_B + \vec{\omega}_{BC} \times \vec{r}_{CB}$$

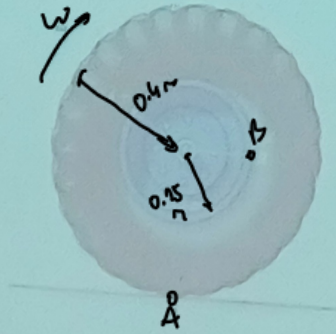
$$2\omega_{AB} \vec{j} = 8(\cos 45^\circ \vec{i} - \sin 45^\circ \vec{j}) - 2\omega_{BC} \vec{i}$$

$$i: 2\omega_{BC} = 8 \cos 45^\circ$$

$$j: 2\omega_{AB} = -8 \sin 45^\circ \quad (2)$$

Rigid body kinematics - 8

If at a given instant, point B has a downward velocity of 3 m/s, determine the velocity of point A at this instant.



$$\vec{v}_B = \vec{v}_{A/B} + \vec{v}_A$$

$$-3\vec{j} = -\omega\vec{k} \times (0.15\vec{i} + 0.4\vec{j}) + \vec{v}_A$$

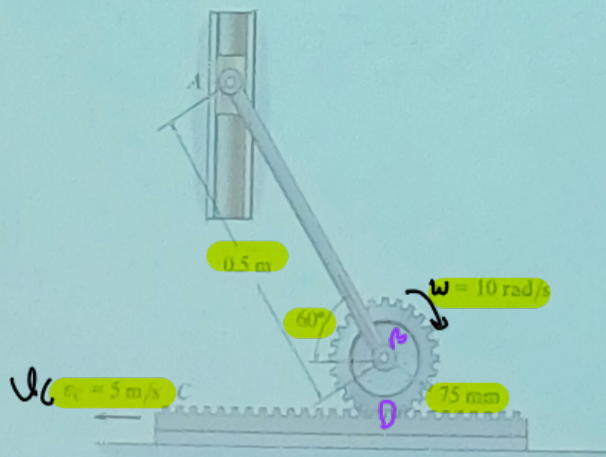
$$-3\vec{j} = -0.15\omega\vec{j} + 0.4\omega\vec{i} + v_A\vec{i}$$

$$i: v_A + 0.4\omega = 0$$

$$j: 0.15\omega = 3 \rightarrow \omega = \frac{3}{0.15} \rightarrow v_A = -0.4 \cdot \frac{3}{0.15} = \boxed{-8}$$

Rigid body kinematics - 9

If the gear rotates with an angular velocity of $\omega = 10 \text{ rad/s}$ and the gear rack moves at $v_C = 5 \text{ m/s}$, determine the velocity of the slider block A at the instant shown.



$$\begin{aligned}
 \vec{v}_A &= \omega_{AB} \times \vec{r}_{BA} = v_{A/B} \quad \text{A, B' gear} \\
 &+ \omega_{BD} \times \vec{r}_{BD} \quad \text{D, D' gear} \\
 &+ v_D = v_C \quad \text{D de rack}
 \end{aligned}$$

$$\begin{aligned}
 \vec{v}_A \hat{j} &= \omega_{AB} \times \vec{r}_{BA} + \omega_{BD} \times \vec{r}_{BD} + v_C \hat{i} \\
 &= \omega_{AB} \hat{k} \times 0.5 (-\cos 60^\circ \hat{i} + \sin 60^\circ \hat{j}) - 10 \hat{k} \times 0.075 \hat{j} - 5 \hat{i} \\
 &= -0.5 \omega_{AB} (\sin 60^\circ \hat{i} + \cos 60^\circ \hat{j}) + 10 \cdot 0.075 \hat{j} - 5 \hat{i}
 \end{aligned}$$

$$i: -0.5 \sin 60^\circ \omega_{AB} + 0.075 - 5 = 0 \rightarrow \omega_{AB} = \dots$$

$$j: v_A = -0.5 \cos 60^\circ \omega_{AB} \rightarrow \boxed{v_A = \dots}$$

~ WEEK 9, LES 2 ~

$$\sum \vec{M}_O = \vec{H}_O$$

$$\vec{H}_O = \sum (\vec{r}_i \times m_i \vec{v}_i)$$

gleichung normieren (denkmal aufpassen sein)

0 nicht sein sondern gleich 0 = Axiom normieren deswegen



G nicht sein sondern gleich 0

$$\vec{H}_G = \sum (\vec{r}_{i/G} \times m_i \vec{v}_i) = \sum [\vec{r}_{i/G} \times m_i (\vec{v}_G + \vec{v}_{i/G})] = \sum (\vec{r}_{i/G} \times m_i \vec{v}_G) + \sum \vec{r}_{i/G} \times m_i \vec{v}_{i/G}$$

G nicht sein sondern gleich 0 sein.

i'mmer gleich 0 sein

$$\vec{r}_G = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

gleich 0 sein

$$\vec{H}_G = \sum \vec{r}_{i/G} \times m_i \vec{v}_{i/G}$$

$$\vec{H}_G = \sum (\vec{r}_{i/G} \times m_i \vec{a}_{i/G}) = \sum (\vec{r}_i \times \vec{F}_i) = \sum \vec{M}_G$$

$$\sum \vec{M}_G = \vec{H}_G$$

$$\vec{H}_O = \int (\vec{r} \times \vec{v}) dm$$

$$\vec{H}_G = \int (\vec{r}_{i/G} \times \vec{v}_{i/G}) dm$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = -\omega^2 \vec{r}$$

$$\vec{H}_O = \int (\vec{r} \times (\vec{\omega} \times \vec{r})) dm = \int (\vec{\omega} r^2) dm = \vec{\omega} \cdot \int r^2 dm = I_O \cdot \vec{\omega}$$

$$\sum \vec{M}_O = I_O \cdot \vec{\alpha}$$

Dann nicht sein sondern

$$I_O = I_G + m d^2$$

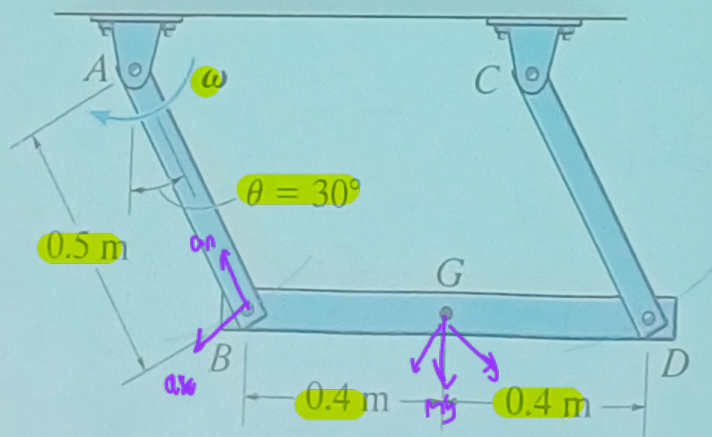
$$\sum \vec{M}_G = I_G \cdot \vec{\alpha}$$

Aber nicht sein sondern

$$\sum \vec{M}_A = I_G \cdot \vec{\alpha} + \vec{r}_{GA} \times m \vec{a}_G = \sum \vec{M}_A = I_A \cdot \vec{\alpha} + \vec{r}_{GA} \times m \vec{a}_A$$

Momenten bei nicht sein sondern

Planar rigid body kinetics-1



The 100-kg beam is supported by two rods having negligible mass. Determine the force developed in each rod for $\theta = 30^\circ$ and $\omega = 6 \text{ rad/s}$.

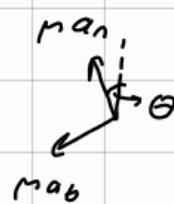
SCD:



$$\sum F_n = m \cdot a_n$$

$$F_{AB} + F_{CD} - mg \cos \theta = m \cdot \frac{v^2}{r}$$

$$= 2 \cdot F_{AB} - mg \cos \theta = m \cdot \omega^2 \cdot r \rightarrow F_{AB} = \dots$$



$$F_{AB} = F_{CD} \text{ dur}$$

$$\sum \vec{M}_G = I_G \cdot \vec{\alpha} = 0$$

↓
kayışın kendisi
dönüyor

↓
Dis kuvvetlerin G noktasına
göre momentleri toplam = 0

$$-F_{AB} \cdot \cos \theta \cdot \frac{L}{2} + F_{CD} \cdot \cos \theta \cdot \frac{L}{2} = 0$$

$$\omega = \frac{v}{r}$$

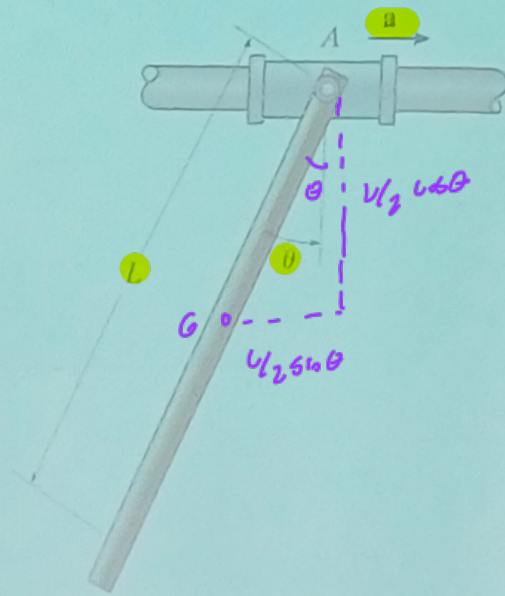
$$a_n = \frac{v^2}{r}$$

Soru yanıtı anca:

$$\sum F_t = m \cdot a_t \rightarrow mg \sin \theta = m \cdot a_t \rightarrow a_t = \dots$$

Planar rigid body kinetics-2

The uniform bar of mass m is pin connected to the collar, which slides along the smooth horizontal rod. If the collar is given a constant acceleration of a , determine the bar's inclination angle θ . Neglect the collar's mass.



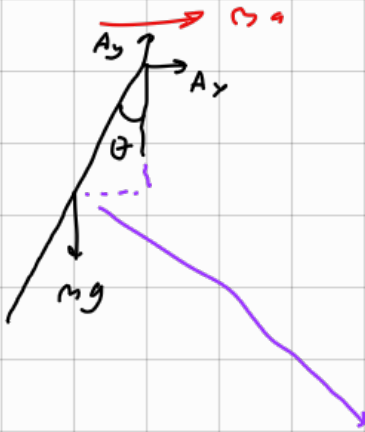
$$\sum \vec{F} = m \vec{g}$$

$$\sum \vec{M}_O = I_O \vec{\alpha}$$

$$\sum \vec{M}_G = I_G \alpha$$

$$\vec{\Sigma M_A} = I_G \cdot \alpha + d m a$$

SLD:



A'ya göre non: $mg \cdot \frac{L}{2} \cdot \sin \theta = ma \frac{L}{2} \cos \theta$

$$\tan \theta = \frac{a}{g} \rightarrow \boxed{\theta = \dots}$$

$$\vec{\epsilon}_{M4} = \cancel{\vec{I}_0 \cdot \vec{a}} + \vec{I}_{6/4} \approx 1.96 \vec{a}$$

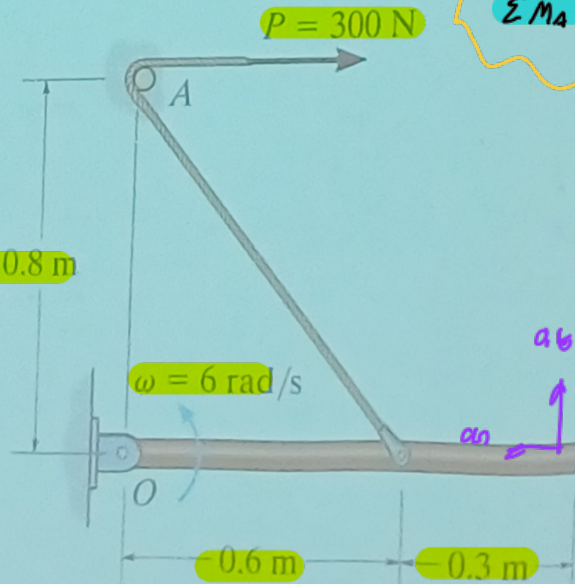
$\cos \alpha = \frac{L}{Z}$

 ağırlık noktasına
 dik uzaklık

~ WEEK 10 ~

Planar rigid body kinetics-3

The uniform 30-kg slender rod is being pulled by the cord. If the rod has a counterclockwise angular velocity of 6 rad/s, determine the reactions at the pin O and the angular acceleration of the rod.

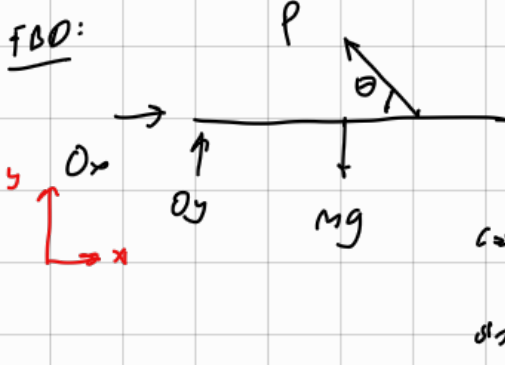


$\Sigma \vec{F} = m \vec{a}$ Heslet
 $\Sigma \vec{M}_O = I_O \cdot \alpha$
 $\Sigma \vec{M}_G = I_G \cdot \alpha$
 $\Sigma \vec{M}_A = I_G \cdot \alpha + d m a$ Dönme

angular acceleration ($\ddot{\theta}$)

$\alpha \cdot r = a_{\text{tangential}}$

$\left. \begin{matrix} \omega^2 r \\ \frac{v^2}{r} \end{matrix} \right\} = a_{\text{normal}}$



$\Sigma M_O = I_O \cdot \alpha \rightarrow -mg \cdot 0.45 + P \cdot \frac{4}{5} \cdot 0.6 = I_O \cdot \alpha$
 $\alpha = \dots$
 $\omega = 6 \text{ rad/s}$
 $a_{\text{tan}} = \omega^2 \cdot r$

$\Sigma F_y = 0 \rightarrow O_y + P \cdot \frac{4}{5} - mg = 0$



$I_G = \int r^2 dm$
 $= \int r^2 \cdot \rho dV$
 $= \int_{-h/2}^{h/2} \int_0^b \int_0^L r^2 \cdot \rho dx dy dz$
 $= \rho b L \int_{-h/2}^{h/2} y^2 dy = \rho b L \cdot \frac{h^3}{12}$
 $m = \rho b h L$

$I_G = \frac{m b^2}{12}$

$I_O = I_G + d^2 m$

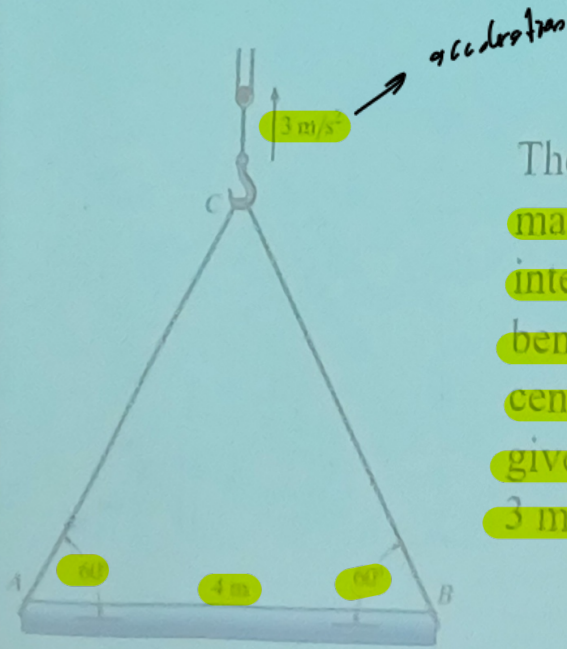
$\Sigma M_O = I_O \cdot \alpha \rightarrow -mg \cdot 0.45 + P \cdot \frac{4}{5} \cdot 0.6 = I_O \cdot \alpha$

$I_O = I_G + d^2 \cdot m$

$I_O = \frac{m L^2}{12} + \left(\frac{L}{2}\right)^2 \cdot m = \frac{m L^2}{3}$

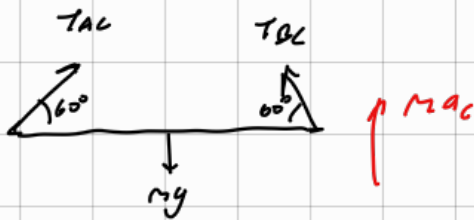
$-mg \cdot 0.45 + P \cdot \frac{4}{5} \cdot 0.6 = \frac{m L^2}{3} \alpha \rightarrow \alpha = \dots$
 $O_y = \dots$
 angular acceleration

Planar rigid body kinetics-4



The uniform girder AB has a mass of 8 Mg . Determine the internal axial, shear, and bending-moment loadings at the center of the girder if a crane gives it an upward acceleration of 3 m/s^2 .

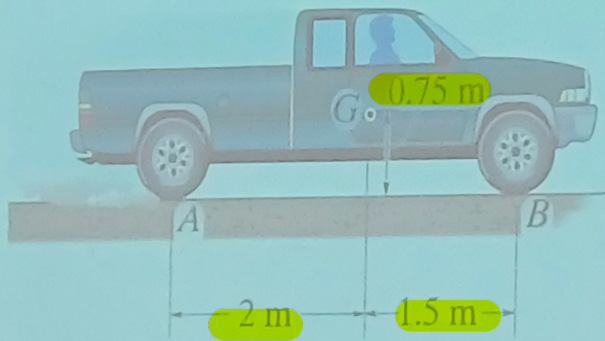
FBD:



$$T_{AC} = T_{BC} = T$$

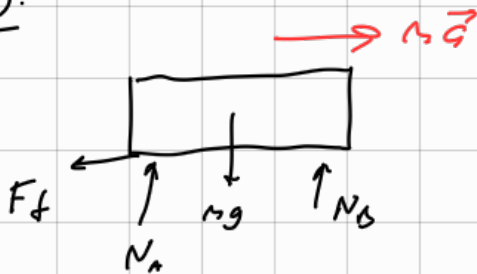
$$\uparrow: 2 \cdot T \cdot \sin 60^\circ - mg = m \cdot a_c \rightarrow \boxed{T = \dots}$$

Planar rigid body kinetics-5



Determine the shortest time possible for the rear-wheel drive, 2-Mg truck to achieve a speed of 16 m/s with a constant acceleration starting from rest. The coefficient of static friction between the wheels and the road surface is 0.8. The front wheels are free to roll. Neglect the mass of the wheels.

FBD:



$$\Sigma F_y = 0 \rightarrow N_A + N_B - mg = 0$$

$$\Sigma M_G = 0 \rightarrow N_B \cdot 1.5 + N_A \cdot 0.75 - mg \cdot 2 = 0$$

$$N_A (\mu \cdot 0.75 - 2 - 1.5) = -mg \cdot 1.5$$

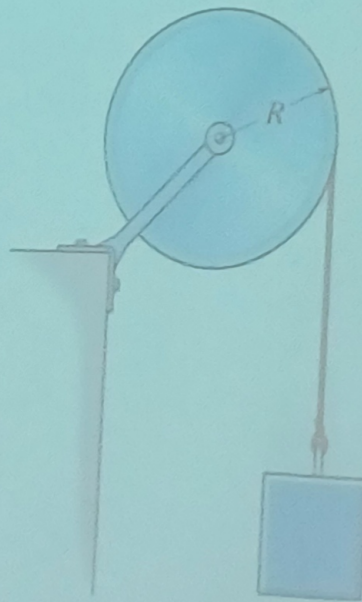
$$N_A = mg \frac{1.5}{2.5 - \mu \cdot 0.75}$$

$$N_B = mg - N_A$$

$$x: F_f = mg \rightarrow N_A \cdot \mu = m \cdot a \rightarrow a = \dots$$

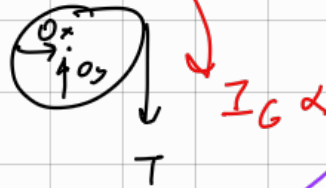
Planar rigid body kinetics-6

The disk has a mass M and a radius R . If a block of mass m is attached to the cord, determine the angular acceleration of the disk when the block is released from rest. Also, what is the velocity of the block after it falls a distance $2R$ starting from rest?



$$I_G = \frac{MR^2}{2}$$

FBD:



$$\sum M_G = I_G \alpha \quad : \quad T \cdot R = I_G \cdot \alpha = \frac{MR^2}{2} \alpha$$

$$\sum F_y = m \cdot a \rightarrow mg - T = ma = m \alpha R$$

$a = \alpha \cdot R$

$$T = m(g - \alpha R)$$

$$\alpha = \frac{2}{3} \frac{g}{R}$$

$$a = \alpha \cdot R = \frac{2}{3} \cdot g$$

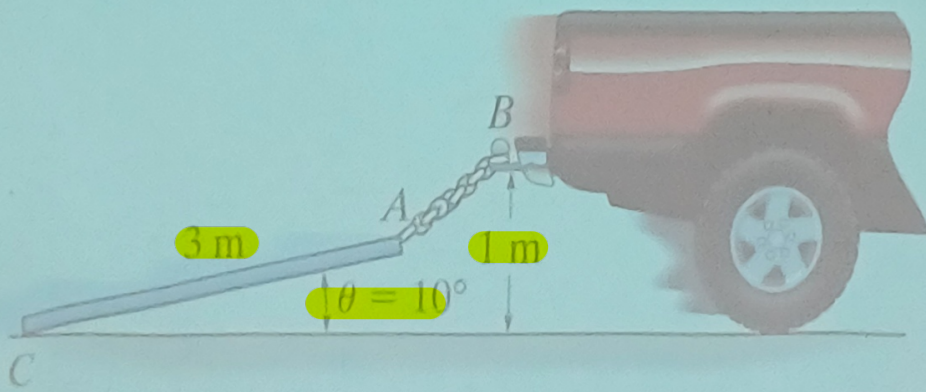
$$v^2 = 2(\alpha R) \cdot (2R)$$

distance

$$v^2 = 2ax$$

because both accelerate

Planar rigid body kinetics-7



The pipe has a length of 3 m and a mass of 500 kg. It is attached to the back of the truck using a 0.6-m-long chain AB. If the coefficient of kinetic friction at C is 0.4, determine the acceleration of the truck if the angle $\theta = 10^\circ$ with the road.



$$\sum F_y = 0 \rightarrow F \sin \alpha - mg + N = 0 \quad (1)$$

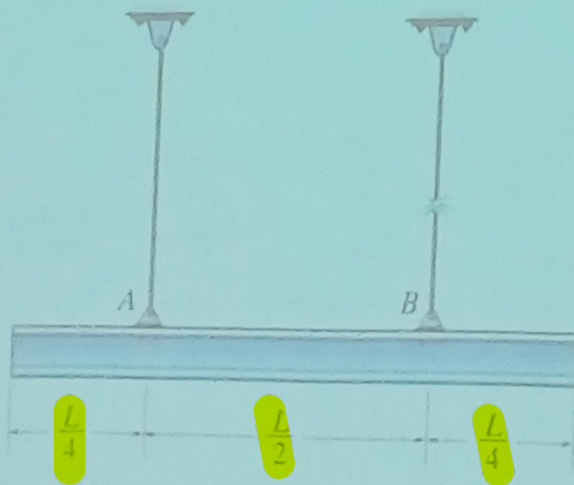
$$\sum F_x = ma \rightarrow F \cos \alpha - N \mu = m \cdot a_c \quad (2)$$

$$\sum M_G = -mg \frac{L}{2} \cos \theta + F (\sin \alpha L \cos \theta - \cos \alpha L \sin \theta) = +FL \sin (\alpha - \theta) \quad (3)$$

$$F = \frac{m \mu (a_c + g)}{(\mu \sin \alpha + \cos \alpha)}$$

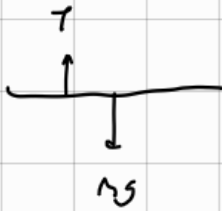
$$\frac{F \cdot \sin (\alpha - \theta)}{m} \cdot \frac{2}{\cos \theta} - g = \underline{\underline{a_c}}$$

Planar rigid body kinetics-10



The uniform beam has a weight W . If it is originally at rest while being supported at A and B by cables, determine the tension in cable A if cable B suddenly fails. Assume the beam is a slender rod.

FBD:



$$\sum M_A = I_A \cdot \alpha$$

$$\frac{3}{4} L W g =$$

$$I_A = I_G + m \left(\frac{L}{5} \right)^2$$

$$= \frac{7}{48} m L^2$$

$$\sum M_G = I_G \cdot \alpha$$

$$7 \frac{1}{4} = \frac{m L^2}{12} \alpha = \frac{12}{7} \frac{3}{4} \frac{m L^2}{12} \rightarrow \boxed{T = \frac{4}{7} W g}$$

~ WEED 10, LES 2 NOT 401 10 ~

Sınav sonu rıd lads → potale dol
Titrigin de ad

WEEK 11

$$\sum \vec{F} = m\vec{a} \rightarrow L = m\vec{U}_G \rightarrow m\vec{U}_{G1} + \int_t \sum \vec{F} dt = m\vec{U}_{G2}$$

$$\sum M_G = I_G \alpha \rightarrow H = I_G \omega \rightarrow I_G \omega_1 + \int_t \sum M_G dt = I_G \omega_2$$

$$\sum M_O = I_O \alpha$$

$$I_G \omega_1 + \int_t \sum M_G dt = I_G \omega_2$$

$$\sum M_A = I_G \alpha + m a_{G0}$$

$$T = \frac{1}{2} m U_G^2 + \frac{1}{2} I_G \omega^2$$

$$T = \frac{1}{2} I_O \omega^2$$

şek. şek.
statistik dizevce

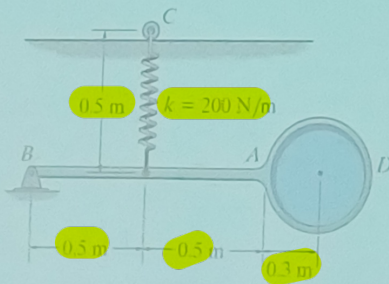
$$U_1 + \int_t \sum M_G dt = U_2$$

TSN
Formüller!

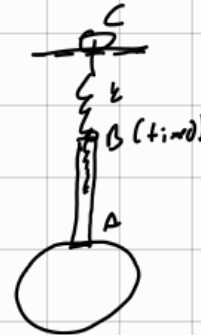
$$T_1 + U_1 = T_2 + U_2$$

$$T_1 + W_{1,2} = T_2$$

Planar rigid body kinetics-13



The pendulum consists of a 6-kg slender rod fixed to a 15-kg disk. If the spring has an unstretched length of 0.2 m, determine the angular velocity of the pendulum when it is released from rest and rotates clockwise 90° from the position shown. The roller at C allows the spring to always remain vertical.



$$T_1 + U_1 = T_2 + U_2$$

sıkma be tön deş ol

$$\frac{1}{2} k \Delta s_1^2 = \frac{1}{2} I_B \omega^2 + \frac{1}{2} k \Delta s_2^2 - (m_1 g \frac{L}{2} + m_2 g (L+R))$$

↓
L şub. → diğ.
işin ası
potansiyel.

$$I_B = \left[m_1 \frac{L^2}{12} + m_1 \left(\frac{L}{2} \right)^2 \right] + \left[m_2 \frac{R^2}{2} + m_2 (L+R)^2 \right]$$

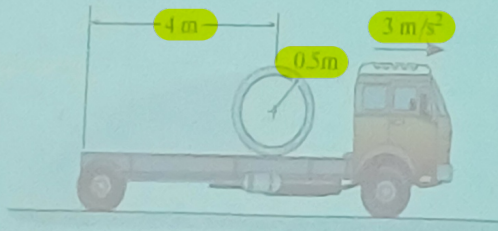
$$I_G = \dots$$

$$\omega = \dots$$

$$T_{\text{disk}} = \frac{1}{2} I \omega^2$$

Özgül kinetik enerji

Planar rigid body kinetics-14

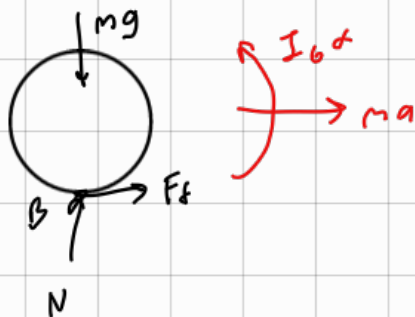


The 500-kg concrete culvert has a mean radius of 0.5 m. If the truck has an acceleration of 3 m/s², determine the culvert's angular acceleration. Assume that the culvert does not slip on the truck bed, and neglect its thickness.

Σ-büt a-tok non.
Dist a-t. non.
İç: boy silindirik a-tok nonati:
Jirasyon vuruşu:
Pestel aban hand

Sinav işin FIZİK

i	j	k	
0	0	α	0 10 10
0	R	0	
i	j	k	-(0 + αR) j
0	0	α	-αR i



$\sum M_B = I_G \cdot \alpha + m g R$

$\downarrow$

$= 0$

zım kuvvetler
B'den geçiyor

Hesapı bu altta yapıyoruz

$\vec{a}_G = \vec{a}_B + \vec{a}_{G/B}$

$= 3\vec{i} + \alpha \vec{k} \times R\vec{j} - \omega^2 R\vec{j}$

3 m/s² a-tok a-tok a-tok

B noktasında a-tok a-tok a-tok
Ciy bilgisi normal vektör
dışarıya a-tok a-tok a-tok

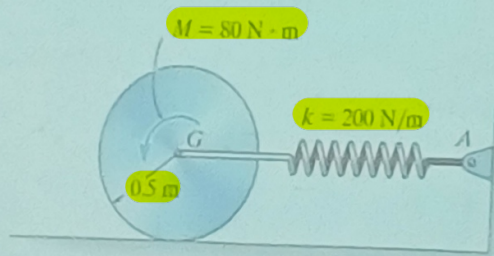
$\vec{a}_G = 3 - 0.5\alpha$

$I_G \cdot \alpha + m \cdot (3 - 0.5\alpha) \cdot R = 0$

$I_G = m R^2$

$\alpha = \dots$

Planar rigid body kinetics-15



The 30-kg disk is originally at rest, and the spring is unstretched. A couple moment of $M = 80\text{ Nm}$ is then applied to the disk as shown. Determine its angular velocity when its mass center G has moved 0.5 m along the plane. The disk rolls without slipping.

$$\cancel{T_1 + V_1 + W_{1-2}} = T_2 + V_2$$

$$W_{1-2} = 80 \cdot \frac{0.5}{0.5}$$

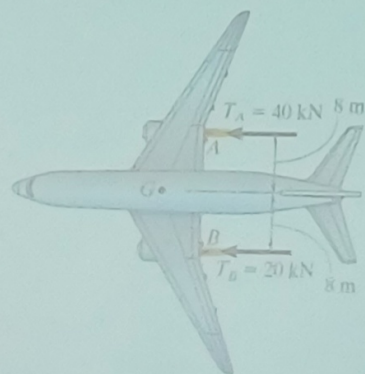
$$\int M d\theta$$

$$R\theta = 0.5 \rightarrow \theta$$

$$80 = \frac{1}{2} I_G \omega^2 + \frac{1}{2} k \Delta s^2 = \frac{1}{2} \left(\frac{3}{2} \cdot 30 \cdot 0.5^2 \omega^2 + \frac{1}{2} \cdot 200 \cdot 0.5^2 \right) \rightarrow \omega$$

$$\frac{1}{2} m R^2 + m R^2 = \frac{3}{2} m R^2$$

Planar rigid body kinetics-16



The airplane is traveling in a straight line with a speed of 300 km/h when the engines A and B produce a thrust of T_A and T_B respectively. Determine the angular velocity of the airplane in $t = 5 \text{ s}$. The plane has a mass of 200 Mg, its center of mass is located at G, and its radius of gyration about G is $k_G = 15 \text{ m}$.

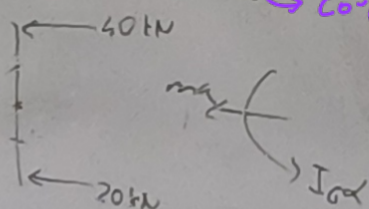
$$H_G = I_G \omega$$

$$H_1 + \int \sum M_G dt = H_2$$

$$I_G \omega + \int_0^5 \sum M_G dt = I_G \omega$$

$$20 \cdot 10^3 \cdot 8 \cdot 5 = 200 \cdot 10^3 \cdot 15^2 \omega$$

$$\text{moment} = (40 - 20) \cdot 10^3 \cdot 8 \rightarrow \text{couple moment}$$



$$\int \sum M_G dt$$

$$\sum M_G = I_G \alpha$$

$$(\sum M_G) \Delta t = I_G \omega$$

$$\text{ang. impulse} = \text{ang. momentum}$$

Planar rigid body kinetics-17



The spool has a mass of 40 kg and a radius of gyration of $k_G = 0.3$ m. If the 10-kg block is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity 15 rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.

m_2

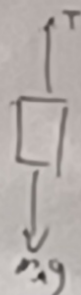
As block
is block

$$T_1 = T_2 = T = T_3$$

$$0 = \left(\frac{1}{2} m_2 v_A^2 + \frac{1}{2} (m_2 k_G^2) \omega^2 \right) - m_2 g h$$

$$\omega R$$

$$0 = \frac{1}{2} 15^2 \left(10 \cdot 0.3^2 + 40 \cdot 0.3^2 \right) - 10 g h$$



$$m_2 g - T = m_2 a$$

$$T = m_2 (g - a)$$

$$v = \omega R$$

$$v = 20 h \rightarrow a \rightarrow T$$

~ WEEK 12 - Titreşim ~ → SON KONU!!

1. SARKAÇ DAVRANIŞI



* Sarkaç hareketi için :
 A) yavaş yavaş hareketler → pot. e. bakandır
 B) hızlı hareketler → kin. e. bakandır

serp:



1

$$t: -mg \sin \theta = mL\ddot{\theta}$$

$$\ddot{\theta} + \frac{g}{L} \sin \theta = 0 \rightarrow \sin \theta \approx \theta \rightarrow \ddot{\theta} + \frac{g}{L} \theta = 0$$



2

$$T + U = \text{const.} \rightarrow \frac{1}{2} mL\dot{\theta}^2 + mgL(1 - \cos \theta) = \text{const.}$$

$$\frac{d}{dt} \rightarrow \ddot{\theta} + \frac{g}{L} \sin \theta = 0$$

3

$$\Sigma M_O = \dot{H}_O \rightarrow -mg \sin \theta L = \frac{d}{dt} (mL^2 \dot{\theta}) = mL^2 \ddot{\theta}$$

4) Lagrange denklemleri

$$L = T + V \rightarrow \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0$$



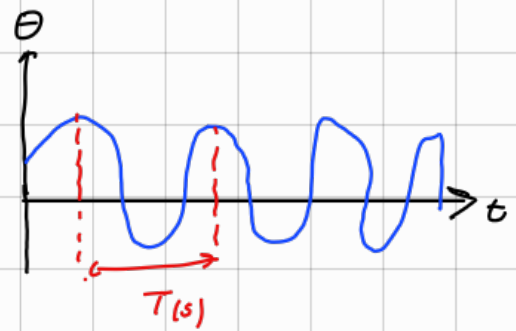
$$\ddot{\theta} + \frac{g}{L} \theta = 0$$



$$\ddot{\theta} + \omega_n^2 \theta = 0$$

$$\omega_n^2 = \frac{g}{L}$$

↙ *angular frequency (rad/s)*



$$f = \frac{1}{T} \quad (Hz = \frac{1}{s})$$

$$\omega_n = \frac{2\pi}{T} \quad (rad/s)$$

$$e^{rt} \rightarrow r^2 + \omega^2 = 0 \rightarrow r = \pm i\omega_n$$

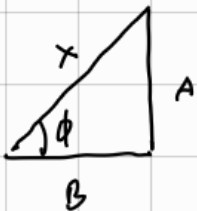
$$\ddot{\theta} + \omega_n^2 \theta = 0$$

$$\theta(t) = A \cos(\omega_n t) + B \cdot \sin(\omega_n t) \quad \left. \vphantom{\theta(t)} \right\} \text{General solution}$$

$$\theta(0) = \theta_0 = A$$

$$\dot{\theta}(0) = \dot{\theta}_0 = B\omega_n \rightarrow B = \frac{\dot{\theta}_0}{\omega_n}$$

$$\theta(t) = \theta_0 \cdot \cos \omega_n t + \frac{\dot{\theta}_0}{\omega_n} \cdot \sin \omega_n t$$



$$\sin \phi = A/X$$

$$\cos \phi = B/X$$

$$X = \sqrt{A^2 + B^2}$$

$$\phi = \tan^{-1}(A/B)$$

$$\theta(t) = X \cdot \sin(\omega_n t + \phi)$$

↙ *amplitude*

↙ *phi*

$$X = \sqrt{\theta_0^2 + \left(\frac{\dot{\theta}_0}{\omega_n}\right)^2}$$

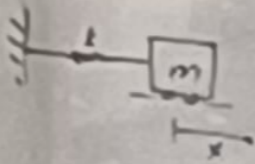
$$\theta(t) = \operatorname{Im} [X \cdot e^{i(\omega_n t + \phi)}]$$

$z = \cos \theta + i \sin \theta$
 $\frac{dz}{d\theta} = -\sin \theta + i \cos \theta = i(\cos \theta + i \sin \theta) = iz \rightarrow \frac{dz}{d\theta} = iz, \frac{dz}{z} = i d\theta \rightarrow z = e^{i\theta} = \cos \theta + i \sin \theta$
 $\frac{d^2 z}{d\theta^2} = -\cos \theta - i \sin \theta = -i^2(\cos \theta + i \sin \theta) = -iz$
 $\sin(\omega_n t) = \frac{1}{2i} (e^{i\omega_n t} - e^{-i\omega_n t})$
 $e^{-i\theta} = \cos \theta - i \sin \theta$
 $\cos \theta = \frac{1}{2} (e^{i\theta} + e^{-i\theta})$
 $\sin \theta = \frac{1}{2i} (e^{i\theta} - e^{-i\theta})$
 $\cosh x = \frac{1}{2} (e^x + e^{-x})$
 $\sinh x = \frac{1}{2} (e^x - e^{-x})$

$\theta(t) = X \sin(\omega_n t + \phi) = \operatorname{Im} [X e^{i(\omega_n t + \phi)}]$

$\sin \phi = A/X$
 $\cos \phi = B/X$
 $X = \sqrt{A^2 + B^2}$
 $\phi = \tan^{-1}(A/B)$

YATGI SİSTEMLER



$$kx \quad \overrightarrow{ma}$$

$\Sigma F = ma \rightarrow -kx = m\ddot{x} \rightarrow m\ddot{x} + kx = 0$

$$\ddot{x} + \omega_n^2 x = 0 \rightarrow x(t) = A \cos \omega_n t + B \sin \omega_n t$$

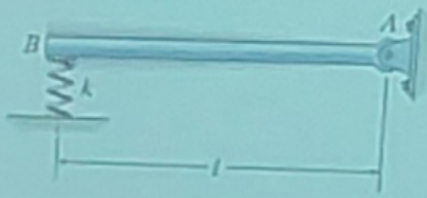
$$\omega_n = \sqrt{\frac{k}{m}}$$

$$x(0) = x_0$$

$$\dot{x}(0) = v_0$$

$$A = x_0, B = \frac{v_0}{\omega_n}$$

Application-1



$$J = L \cdot \sin \theta$$

$$\Sigma M_A = I_A \ddot{\theta} \rightarrow -kl \sin \theta L \cos \theta - mg \frac{L}{2} \cos \theta = \frac{1}{3} L^2 \ddot{\theta}$$

Moment

$$\ddot{\theta} + \frac{3k}{L} \sin \theta \cos \theta + \frac{3}{2} \frac{g}{L} \cos \theta = 0$$

$$\sin \theta \approx \theta, \cos \theta \approx 1$$

$$\ddot{\theta} + \frac{3k}{L} \theta = -\frac{3}{2} \frac{g}{L}$$

$$\theta(t) = \theta_h + \theta_p$$

$$\theta_p = \frac{-\frac{3}{2} \frac{g}{L}}{\frac{3k}{L}} = \frac{-g}{2kl}$$

$$\theta - \theta_p$$

$$\theta' + \frac{3k}{L} \theta = \frac{3}{2} \frac{g}{L}$$

$$\theta' + \frac{3k}{L} \theta = 0$$

stetigen
funkt. wdh.

$$\omega_n = \sqrt{\frac{3k}{L}}$$

anfangs- und endgeschw. = 0

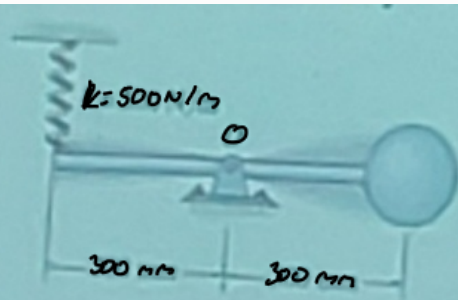
Energie: Konstante $\rightarrow T + V = \text{const.}$

$$\frac{d}{dt} \left[\frac{1}{2} I_0 \dot{\theta}^2 + mg \frac{L}{2} \sin \theta + \frac{1}{2} k (L \sin \theta)^2 \right] = 0$$

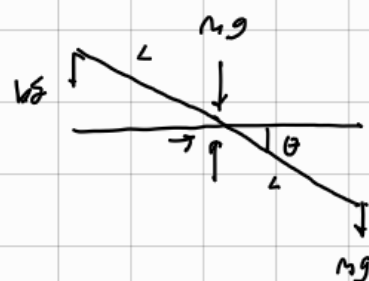
$$\frac{mL^2}{2} \cdot \ddot{\theta} + mg \frac{L}{2} \cos \theta \cdot \dot{\theta} + k L^2 \sin \theta \cos \theta \cdot \dot{\theta} = 0$$

$$\ddot{\theta} + \frac{3k}{L} \theta = \frac{3}{2} \frac{g}{L}$$

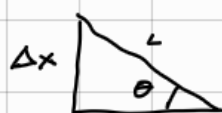
$$\omega_n = \sqrt{\frac{3k}{L}}$$



Find the vibration period for $m = 3 \text{ kg}$.



$$T + V = \text{const.}$$



$$\Delta x = L \cdot \sin \theta$$

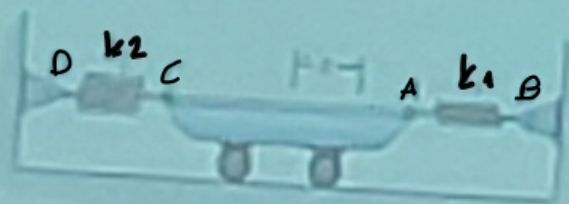
$$\frac{1}{2} m (L\dot{\theta})^2 + \frac{1}{2} k (L \cdot \sin \theta)^2 = \text{const.}$$

$$\frac{d}{dt} = m L^2 \dot{\theta} \ddot{\theta} + k L^2 \sin \theta \cos \theta \dot{\theta} = 0 \rightarrow \ddot{\theta} + \frac{k}{m} \sin \theta \cos \theta = 0 \rightarrow \sin \theta \approx \theta, \cos \theta \approx 1 \rightarrow \ddot{\theta} + \frac{k}{m} \theta = 0$$

$$\omega_n = \frac{k}{m}$$

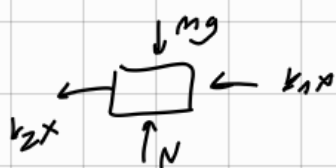
$$T = \frac{2\pi}{\omega_n} = \sqrt{\frac{2\pi m}{k}}$$

$$\omega_n = 2\pi f \quad \left. \begin{array}{l} f = \frac{1}{T} \\ T = \frac{2\pi}{\omega_n} \end{array} \right\}$$



Find the natural frequency.

1



$$\sum F = ma \rightarrow -(k_1 + k_2)x = m\ddot{x}$$

$$m\ddot{x} + (k_1 + k_2)x = 0$$

$$\ddot{x} + \frac{k_1 + k_2}{m} x = 0$$

$$\omega_n = \sqrt{\frac{k_1 + k_2}{m}}$$

2

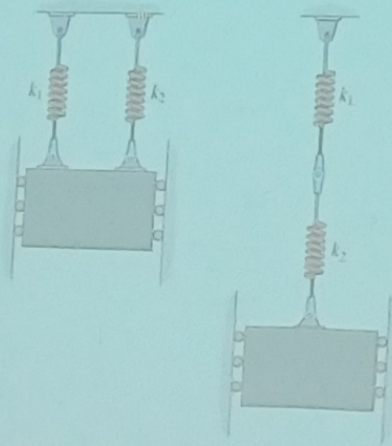
$$T + V = \text{const.}$$

$$\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k_1 x^2 - \frac{1}{2} k_2 x^2 = \text{const.}$$

$$\frac{d}{dt} \rightarrow \dots \rightarrow$$

$$\omega_n = \sqrt{\frac{k_1 + k_2}{m}}$$

Application-2



The 15-kg block is suspended from two springs having a different stiffness and arranged parallel to each other, and as a series. If the natural periods of oscillation of the parallel system and series system are observed to be 0.5 s and 1.5 s, respectively, determine the spring stiffnesses k_1 and k_2 .

$\begin{array}{c} k_1 \\ \text{---} \\ k_2 \end{array} \rightarrow F = \text{---} \rightarrow F$
 $k_3 = k_1 + k_2$

$\text{---} \rightarrow F$
 $\begin{array}{c} k_1 \\ \text{---} \\ k_2 \end{array} = \text{---} \rightarrow F$
 $\frac{1}{k_3} = \frac{1}{k_1} + \frac{1}{k_2}$

$$\omega_1^2 = \frac{k_1 + k_2}{m} = \left(\frac{2\pi}{0.5} \right)^2$$

$$\omega_2^2 = \frac{k_1 k_2}{m(k_1 + k_2)} = \left(\frac{2\pi}{1.5} \right)^2$$

$$\omega = \frac{2\pi}{T}$$

$$\left. \begin{array}{l} k_1 = \dots \\ k_2 = \dots \end{array} \right\}$$

~ WEEK 13 ~ Notional

~ WEEK 13, LES 2 ~

$$m\ddot{x} + kx = F_0 \cos \omega t$$

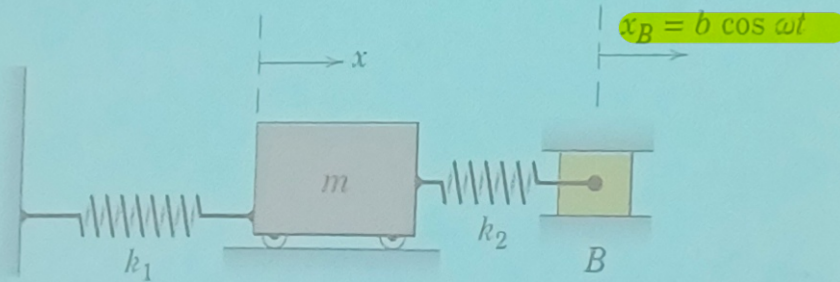
$$x(t) = X_0 \beta \cos \omega t$$

$$X = \frac{F}{k}$$

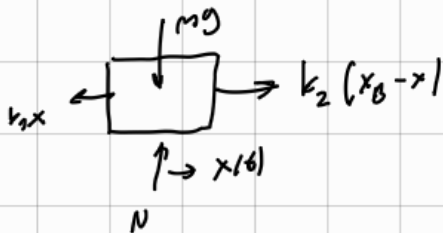
$$\beta = \frac{1}{1-r^2}$$

$$r = \frac{\omega}{\omega_0}$$

Application-6



Attachment B is given a horizontal motion $x_B = b \cos \omega t$. Derive the equation of motion for the mass m and state the critical frequency ω_c for which the oscillations of the mass become excessively large.



$$\sum F = ma$$

$$k_2(x_B - x) - k_1x = m\ddot{x}$$

$$m\ddot{x} + (k_1 + k_2)x = k_2x_B$$

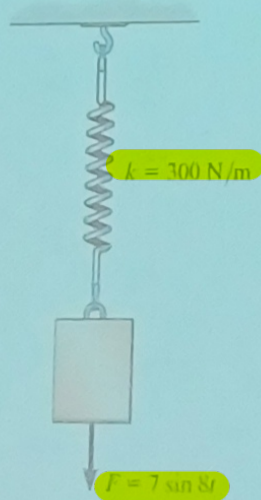
$$m\ddot{x} + (k_1 + k_2)x = k_2b \cos \omega t$$

separieren $\rightarrow X(t) = X_h + X_s \rightarrow$ ges. + d. ges.

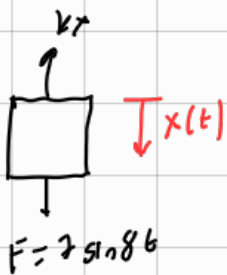
$$= A \cos \omega_n t + B \sin \omega_n t + X_0 \beta \cos \omega t$$

$$\omega_n = \frac{\sqrt{k_1 + k_2}}{m}, \quad X_0 = \frac{k_2 b}{k_1 + k_2}, \quad \beta = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

Application-7



A 5-kg block is suspended from a spring having a stiffness of 300 N/m. If the block is acted upon by a vertical force $F = 7 \sin 8t$, determine the equation that describes the motion of the block when it is pulled down 100 mm from the equilibrium position and released from rest at $t = 0$.



$$\sum \vec{F} = m\vec{a}$$

$$m\ddot{x} = 7 \sin 8t - kx$$

$$5\ddot{x} + 300x = 7 \sin 8t$$

$$x(t) = \underbrace{A \cos \omega_n t + B \sin \omega_n t}_{\text{homogeneous solution}} + \underbrace{X_s \beta \sin \omega t}_{\text{particular solution}}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{300}{5}}, \quad X_s = \frac{F}{k} = \frac{7}{300}, \quad r = \frac{\omega}{\omega_n} = \frac{8}{\sqrt{300/5}}$$

$$\beta = \frac{1}{1-r^2} = \dots$$

$$x(0) = 0.1 \text{ m}$$

$$\dot{x}(0) = 0$$

$$x(t) = \omega_n [-A \sin \omega_n t + B \cos \omega_n t] + X_s \beta \cos \omega t$$

$$\dot{x}(0) = \omega_n B + X_s \beta \omega = 0 \rightarrow \underline{\underline{B}} = \frac{X_s \beta \omega}{\omega_n} = \boxed{\dots} \rightarrow \boxed{A = \dots}$$

$$x(t) = A \cos \omega_n t + B \sin \omega_n t + X_s \beta \sin \omega t$$

$$m \ddot{x} + kx = F \sin \omega t \quad \longleftrightarrow \quad \ddot{x} + \omega_n^2 x = X_s \omega_n^2 \sin \omega t$$

LAPLACE Transformations:

$$X(s) = \mathcal{L}\{x(t)\}$$

$\downarrow$ Laplace domain
 $\downarrow$ zero domain

1) Domain: $t \rightarrow s$

2) Inverse: $x(s) = \dots$

3) Tes Domain: $s \rightarrow t$

$$\mathcal{L}\{\ddot{x}(t)\} = -x(0) - s x(0) + s^2 X(s)$$

$$\mathcal{L}\{\sin \omega t\} = \frac{\omega}{s^2 + \omega^2}$$

(1) $\rightarrow$ Domain

$$-\ddot{x}(0) - s x(0) + s^2 X(s) - \omega_n^2 X(s) = X_s \omega_n^2 \frac{\omega}{s^2 + \omega^2}$$

(2) $\rightarrow$ Inverse

$$X(s) (s^2 - \omega_n^2) = X_s \omega_n^2 \frac{\omega}{s^2 + \omega^2} + V_0 + s X_0$$

$$X(s) = X_s \omega_n^2 \frac{\omega}{(s^2 + \omega^2)(s^2 - \omega_n^2)} + \frac{V_0}{s^2 + \omega_n^2} + X_0 \frac{s}{s^2 + \omega_n^2}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2 + \omega_n^2}\right\} = \cos \omega_n t$$

$$\mathcal{L}^{-1}\left\{\frac{s^2}{s^2 + \omega_n^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{\omega_n} \frac{\omega_n}{s^2 + \omega_n^2}\right\} = \frac{1}{\omega_n} \mathcal{L}^{-1}\left\{\frac{\omega_n}{s^2 + \omega_n^2}\right\} = \frac{\sin \omega_n t}{\omega_n}$$

3) $\rightarrow$ Tes Domain

$$X(s) = X_s \omega_n^2 \frac{\omega}{(s^2 + \omega^2)(s^2 - \omega_n^2)} + \frac{V_0}{s^2 + \omega_n^2} + X_0 \frac{s}{s^2 + \omega_n^2}$$

$\frac{V_0 \sin \omega_n t}{\omega_n}$ $X_0 \cos \omega_n t$ $X(0) = V_0$
 $\uparrow$ $\downarrow$ $\ddot{x}(0) = V_0$

$$P(s)Q(s) = \frac{As+B}{s^2+\omega^2} + \frac{Cs+D}{s^2+\omega_n^2} = \frac{1}{\omega_n^2-\omega^2} \left(\frac{1}{s^2+\omega^2} - \frac{1}{s^2+\omega_n^2} \right) \Rightarrow \mathcal{L}^{-1} \left[\frac{1}{(s^2+\omega^2)(s^2+\omega_n^2)} \right] = \frac{1}{\omega_n^2-\omega^2} \left(\frac{\sin \omega t}{\omega} - \frac{\sin \omega_n t}{\omega_n} \right)$$

$$1 = (A+B)(s^2+\omega_n^2) + (Cs+D)(s^2+\omega^2)$$

$$= s^2(A+B) + s^2(B+D) + s(Cs+D) + B\omega_n^2 + D\omega^2$$

$$\begin{cases} s^2: A+B=0 \\ s^2: B+D=0 \\ s: C+D=0 \\ \text{const: } B\omega_n^2 + D\omega^2 = 1 \end{cases} \rightarrow A=C=0$$

$$s^0: B\omega_n^2 + D\omega^2 = 1 \rightarrow B(\omega_n^2 - \omega^2) = 1 \rightarrow B = -D = \frac{1}{\omega_n^2 - \omega^2}$$

$$X(t) = \mathcal{L}^{-1}[X(s)] = X_{0,11} \frac{1}{\omega_n^2 - \omega^2} \left(\frac{\sin \omega t}{\omega} - \frac{\sin \omega_n t}{\omega_n} \right) + \frac{V_0}{\omega_n} \sin \omega_n t + X_0 \cos \omega t$$

$$\boxed{X(s) = \frac{1}{1-r^2} \left(\frac{1}{s^2+\omega^2} - \frac{1}{s^2+\omega_n^2} \right) + \frac{V_0}{\omega_n} \sin \omega_n t + X_0 \cos \omega t} \quad \text{SON!}$$

burası koda dahil edilmisli;

FREE DAMPED VIBRATION

$$m\ddot{x} + c\dot{x} + kx = 0$$

-işletme sınırları

C: sönüm katsayısı

esajı etile karıştır.

$$mr^2 + cr + k = 0 \rightarrow r_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

$$c_{cr} = 2\sqrt{mk}$$

$$(k = \omega_n \cdot m)$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\frac{c}{m} = 2\zeta\omega_n$$

Sönüm oranı

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

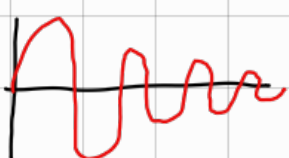
$$r_{1,2} = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$$\zeta = 1 \rightarrow r_{1,2} = -\zeta\omega_n \rightarrow x(t) = e^{-\zeta\omega_n t} (A + Bt)$$

$$\zeta > 1 \rightarrow r_{1,2} \rightarrow x(t) = A \cdot e^{r_1 t} + B \cdot e^{r_2 t}$$

$$\zeta < 1 \rightarrow x(t) = e^{-\zeta\omega_n t} (A \cos \omega_d t + B \sin \omega_d t)$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$



$$x(0) = x_0, \quad \dot{x}(0) = v_0$$

$$x(0) = A = x_0$$

$$\dot{x}(t) = e^{-\gamma t} \left(-\gamma \cos(\omega_d t) + \omega_d \sin(\omega_d t) \right) + \omega_d \left(-A \sin(\omega_d t) + B \cos(\omega_d t) \right)$$

$$\dot{x}(0) = -\gamma A + \omega_d B = v_0 \Rightarrow B = \frac{v_0 + \gamma x_0}{\omega_d}$$

$$\gamma < 1$$

$$A = x_0, \quad B = \frac{v_0 + \gamma x_0}{\omega_d}$$

$$x(0) = x_0, \quad \dot{x}(0) = v_0$$

$$x(0) = A = x_0$$

$$\dot{x}(t) = e^{-\gamma t} \left(-\gamma \cos(\omega_d t) + \omega_d \sin(\omega_d t) \right) + \omega_d \left(-A \sin(\omega_d t) + B \cos(\omega_d t) \right)$$

$$\dot{x}(0) = -\gamma A + \omega_d B = v_0 \Rightarrow B = \frac{v_0 + \gamma x_0}{\omega_d}$$

$$\gamma < 1$$

$$A = x_0, \quad B = \frac{v_0 + \gamma x_0}{\omega_d}$$

$$\gamma < \frac{c}{2m\omega_n}$$

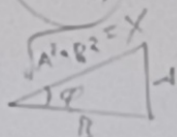
$$x(t) = e^{-\gamma t} (A \cos(\omega_d t) + B \sin(\omega_d t))$$

$$= x_0 e^{-\gamma t} (\cos(\omega_d t) + \frac{\gamma x_0 + v_0}{\omega_d} \sin(\omega_d t))$$

$$x(t) = x_0 e^{-\gamma t} \sin(\omega_d t + \phi)$$

$$x(t) = x_0 e^{-\gamma t} \sin(\omega_d t + \phi)$$

$$\frac{x(t)}{x(t+\tau)} = e^{-\gamma \tau} \frac{\sin(\omega_d t + \phi)}{\sin(\omega_d (t+\tau) + \phi)} \rightarrow \frac{x(t)}{x(t+\tau)} = \frac{1}{e^{\gamma \tau}}$$



$$\sin \phi = \frac{B}{1}$$

$$\cos \phi = \frac{A}{1}$$

$$\delta^2 (1 - \gamma^2) = (2\pi)^2 \gamma^2$$

$$\gamma^2 ((2\pi)^2 + \delta^2) = \delta^2$$

$$\gamma = \frac{\delta}{\sqrt{\delta^2 + (2\pi)^2}} \Rightarrow \gamma \propto \frac{2\pi m \omega_n \delta}{\sqrt{\delta^2 + (2\pi)^2}}$$

~ WEEK 14, LES 1 ~

~ WEEK 14, LES 2 ~

Harmonik Zwangs Actives Systeme Treppen

$$m\ddot{x} + c\dot{x} + kx = F e^{i\omega t} \quad \text{cosub + i sinub (euler)}$$

$$\ddot{x} + \frac{c}{m} \dot{x} + \omega_n^2 x = \frac{F}{m} e^{i\omega t}$$

$\frac{c}{2m\omega_n} = \zeta$ $\frac{F}{m} = \frac{F}{k} \cdot \frac{k}{m} = x_s \omega_n^2$

ζ Cor

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = x_s \omega_n^2 e^{i\omega t} \rightarrow x(t) = x_h + x_p \rightarrow \underline{x_p(t) = A e^{i\omega t}}$$

$\downarrow$ $\downarrow$
Stabilität $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
dini

$$A \cdot (\omega^2 + 2\zeta\omega\omega_n + \omega_n^2) = x_s \omega_n^2 \rightarrow A = x_s \cdot \frac{1}{(1-r^2) + 2i\zeta r} \quad r = \frac{\omega}{\omega_n}$$

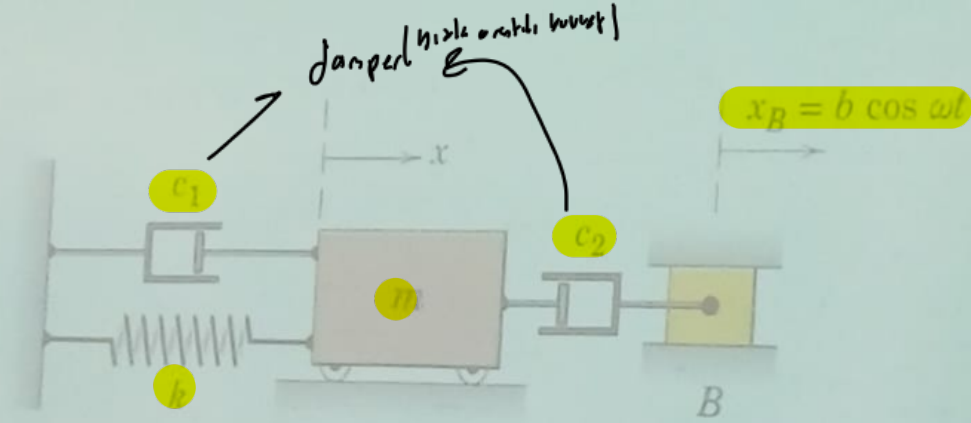
$$x_{p(t)} = x_s \cdot B e^{i\omega t} \rightarrow x_p(t) = \frac{1}{[(1-r^2)^2 + 2\zeta r^2]^{1/2}} \cdot e^{i(\omega t - \psi)} \quad \psi = \tan^{-1} \left(\frac{2\zeta r}{1-r^2} \right)$$

Stabilität + dini

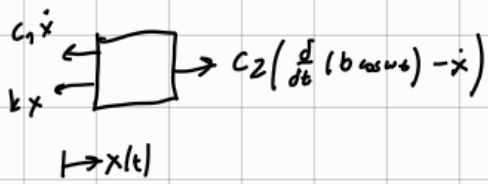
$$x_t(t) = e^{-\zeta\omega_n t} \cdot x \cdot \sin(\omega_n t) + F_s B e^{i\omega t}$$

$\downarrow$

$$e^{i\omega t} (A \cos \omega t + B \sin \omega t)$$



Attachment B is given a horizontal motion $x_B = b \cos \omega t$. Derive the equation of motion for the mass m and state the critical frequency for which the oscillations of the mass become excessively large. What is the damping ratio ζ for the system?



$$\sum F = ma \rightarrow -c_1 \dot{x} - kx + c_2 \left(\frac{d}{dt}(b \cos \omega t) - \dot{x} \right) = m \ddot{x}$$

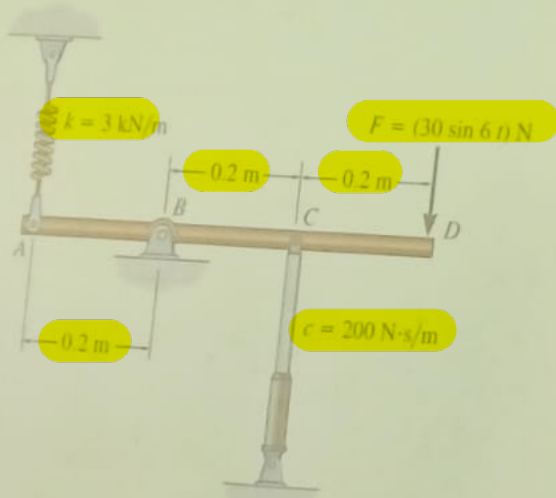
$$m \ddot{x} + (c_1 + c_2) \dot{x} + kx = -c_2 \cdot b \cdot \omega \cdot \sin \omega t$$

Hooke's law.

$$\omega_n = \sqrt{\frac{k}{m}} \rightarrow c_{cr} = 2m\omega_n = 2\sqrt{mk} \rightarrow \zeta = \frac{c_1 + c_2}{2\sqrt{mk}} \quad \zeta = \frac{c}{c_{cr}}$$

$$\omega = \omega_{res} = \omega_n = \text{---}$$

Application-16



If the 12-kg rod is subjected to a periodic force of $F = 30 \sin 6t$, determine the steady state vibration amplitude of the rod.