

~ TEK SERBESTLİK DEREJELİ ~

• $ma = -kx \rightarrow m\ddot{x} + kx = 0 \rightarrow \ddot{x} + \omega_n^2 x = 0$

$\omega_n = \sqrt{\frac{k}{m}}$

$f_n = \frac{\omega_n}{2\pi}$

$T = \frac{2\pi}{\omega_n}$

• $y_n(t) = C \cdot \sin(\omega_n t - \phi)$

$C = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2}$

x_0 : Başlangıç konumu

$\dot{x}_0$: Başlangıç hızı

$\dot{x}_{max} = \omega_n \cdot x_0$

↳ max hız

$\phi = \tan^{-1}\left(\frac{\dot{x}_0 \omega_n}{x_0}\right)$

C: Genlik



$k = F = \frac{\pi d^2}{4} \sigma$

• δ_{st}

$\sum F_x = 0 \rightarrow -3\delta_{st} + mgsin\theta$

$\sum F_x = m\ddot{x} \rightarrow \ddot{x} + \omega_n^2 x = 0$

$\ddot{x} + \omega_n^2 x = 0$

~ İKİ SERBESTLİK DEREJELİ ~

① $\begin{cases} m_1 \ddot{x}_1 = \dots \\ m_2 \ddot{x}_2 = \dots \end{cases}$

② $\begin{cases} x_1(t) = X_1 \sin \omega_n t \rightarrow \ddot{x}_1(t) = -\omega_n^2 X_1 \sin \omega_n t \\ x_2(t) = X_2 \sin \omega_n t \rightarrow \ddot{x}_2(t) = -\omega_n^2 X_2 \sin \omega_n t \end{cases}$

③ $\left(\frac{x_1}{x_2}\right)_1 = \dots \quad \left(\frac{x_1}{x_2}\right)_2 = \dots \quad = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

④ $\left(\frac{x_1}{x_2}\right)_1 = \left(\frac{x_1}{x_2}\right)_2 \rightarrow \dots \quad (\omega_n^2 = \lambda) \rightarrow \lambda_{1,2} = \dots \quad \begin{matrix} \lambda_1 = \dots \\ \lambda_2 = \dots \end{matrix}$

⑤ $\begin{cases} \omega_{n1} = \dots \rightarrow \left(\frac{x_1}{x_2}\right)_1 = \dots \quad x_1 = 1 \rightarrow x_2 = \dots \\ \omega_{n2} = \dots \rightarrow \left(\frac{x_1}{x_2}\right)_2 = \dots \quad x_1 = 1 \rightarrow x_2 = \dots \end{cases}$

~ LAGRANGE ~

1. TEK SERBESTLIK DEGRESI:

① $T = \dots (\dot{\theta}) \rightarrow$
 $V = \dots (\theta)$

$u = w \cdot r = \dot{\theta} \cdot l$

$T = \frac{1}{2} I \omega^2 = \frac{1}{2} I \dot{\theta}^2$

② $L = T - V = \dots (\dot{\theta}, \theta)$

$\frac{\partial L}{\partial \dot{\theta}} = \dots \rightarrow \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = \dots, \frac{\partial L}{\partial \theta} = \dots$

$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0 \rightarrow \dots$

$\cos' \rightarrow -\sin$
 $\sin' \rightarrow \cos$

③ $\ddot{\theta} + \underbrace{\dots}_{\omega_n^2} \theta = 0 \rightarrow \boxed{\omega_n = \dots}$

2. İKİ SERBESTLİK DEGRESI:

① $T = \dots (\dot{\theta}_1, \dot{\theta}_2)$
 $V = \dots (\theta_1, \theta_2)$

② $L = T - V = \dots$

2a) $\frac{\partial L}{\partial \dot{\theta}_1} = \dots \rightarrow \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) = \dots, \frac{\partial L}{\partial \theta_1} = \dots$

$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = 0 \rightarrow \dots$ (1)

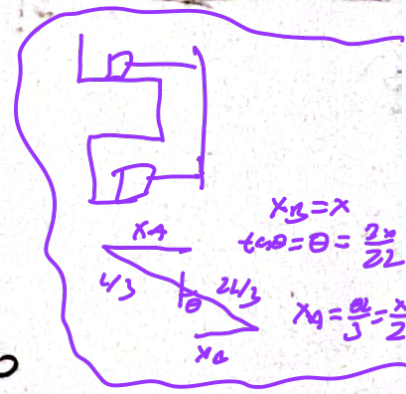
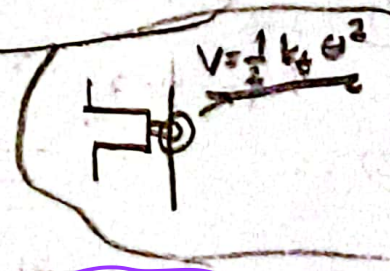
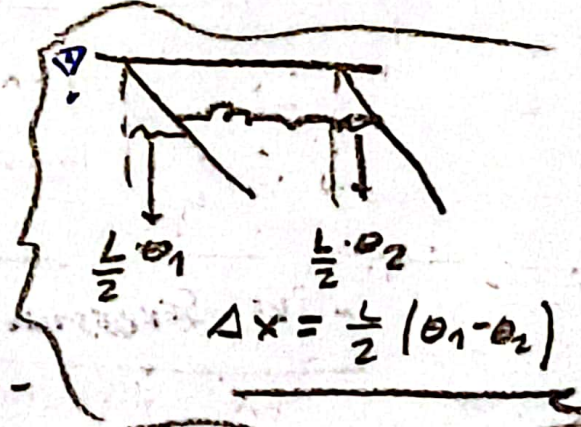
2b) $\frac{\partial L}{\partial \dot{\theta}_2} = \dots \rightarrow \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) = \dots, \frac{\partial L}{\partial \theta_2} = \dots$

$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = 0 \rightarrow \dots$ (2)

③ $\theta_1(t) = A \cdot \cos \omega_1 t \rightarrow \dot{\theta}_1 = -\omega_1 A \cdot \sin \omega_1 t$ (3)
 $\theta_2(t) = B \cdot \cos \omega_2 t \rightarrow \dot{\theta}_2 = -\omega_2 B \cdot \sin \omega_2 t$ (4)

(1), (3) $\rightarrow \dots (\theta_1, \dot{\theta}_1)$
 (2), (4) $\rightarrow \dots (\theta_2, \dot{\theta}_2)$ } $\rightarrow \begin{bmatrix} \dots \\ \dots \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = 0$

$= 0 \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \underline{a \cdot d - c \cdot b = 0}$



7- TEK NODLU SİSTEMLER ~ HOLTZER METODU ~

①



$$\omega_n = \sqrt{\frac{K_4 (J_4 + J_5)}{J_4 \cdot J_5}}$$

②

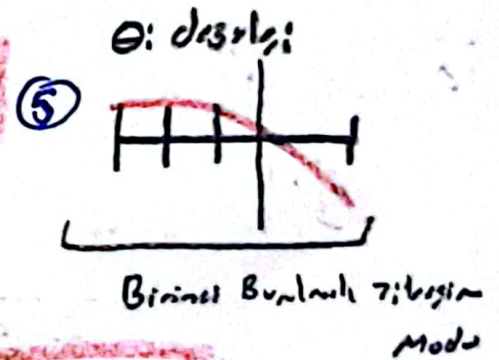
No	J_i	$J_i \omega_n^2$	θ_i	$T_i = J_i \omega_n^2 \theta_i$	$\sum T_i$	K_i	Bu, lma $\Delta \theta = \frac{\sum T_i}{K_i}$
1			$1 - \Delta \theta_1$				

③

$\frac{T_{res}}{a} = \frac{\omega_n}{b}$ → $\frac{x-b}{d-b} = \frac{c-0}{c-a}$ → $x = \omega_n$

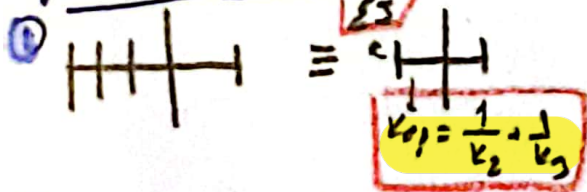
④

$\rightarrow \omega_n \cdot 0.95$
 $\rightarrow \omega_n \cdot 1.05$



4- Tekrar Tablo

9- İki NODLU SİSTEMLER



$$\omega_n^2 = K_e \cdot \frac{J_4 + (J_1 + J_2 + J_3)}{J_4 \cdot (J_1 + J_2 + J_3)}$$

② Tablo → $\sum T_i$

$\rightarrow \omega_n \cdot 1.05$
 $\rightarrow \omega_n \cdot 0.95$

③ iterasyonla $T_{res} = +0.2 \cdot 10^6$ sibi değış değışe gelir

$\omega_n = \dots$

④

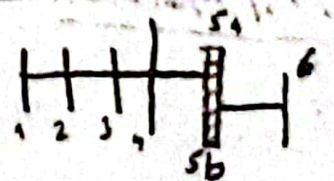
θ_i değerleri } ikinci burunlu titreşim modu

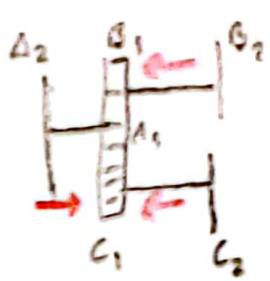
2 yerde 0'ı keser → 2 Nodlu

5- Dişli Kutusu

No	J_i	$J_i \omega_n^2$	θ_i	$T_i = J_i \omega_n^2 \theta_i$	$\sum T_i$	K_i	$\Delta \theta = \frac{\sum T_i}{K_i}$
1			1.0				
2							
3							
4							
5a	-	-	θ_{5a}	-	$\sum T_{5a}$		
5b	-	-	$1.0 \theta_{5a}$	-	$\frac{\sum T_{5a}}{r} + T_{5b}$		
6	-	-	-	-	-		

$$r = \frac{N_{sb}}{N_{sa}}$$





① $\frac{N_0}{A_2} = \frac{J_1}{A_1} = \frac{J_2 \cdot \omega_1 \cdot \theta_1}{\Delta A_1} = \frac{T_1 = J_1 \cdot \omega_1 \cdot \theta_1}{\Delta A_1} = \frac{\sum T_i}{\Delta A_1} = \frac{\sum T_i}{\Delta A_1} = \frac{\Delta \theta = \frac{\sum T_i}{k_i}}{\Delta A_1}$

② $B_2 = \beta$
 $B_1 = \Delta B_1$

$\Delta B_1 = r \cdot \Delta A_1 \rightarrow \beta = \dots$ $\rightarrow \sum T_{B1} \rightarrow T_B = r \cdot \sum T_{B1}$

③ $C_2 = \gamma$
 $C_1 = \Delta C_1$

$\Delta C_1 = r \cdot \Delta A_1 \rightarrow \gamma = \dots$ $\rightarrow \sum T_{C1} \rightarrow T_C = r \cdot \sum T_{C1}$

④ $T_A + T_B + T_C = 0$ olmalı

SÖNÜMLÜ SERBEST TIT.

1. ($\zeta < 1$) Zayıf Sönüm

① $\zeta = \frac{c}{2\sqrt{km}} < 1 \rightarrow$ Zayıf sönüm

② $\omega_n = \sqrt{\frac{k}{m}} \rightarrow \omega_d = \omega_n \sqrt{1 - \zeta^2}$
serbest dalga sönümlü dalga

③ Başlangıç Koşulları $\bar{A} = \frac{(\dot{x}_0 + \zeta \omega_n x_0)^2 + (x_0 \omega_d)^2}{\omega_d^2}$ $\phi = \tan^{-1} \frac{x_0 \omega_d}{\dot{x}_0 + \zeta \omega_n x_0}$

④ Herbat 0. anı $x(t) = \bar{A} \cdot e^{-\zeta \omega_n t} \cdot \sin(\omega_d t + \phi)$
 $x_0 = \bar{A} \cdot \sin \phi$
 $\dot{x}_0 = -\zeta \omega_n \bar{A} \sin \phi + \omega_d \bar{A} \cos \phi$

2. ($\zeta > 1$) Ağır Sönüm

① $\zeta = \frac{c}{2\sqrt{km}} > 1 \rightarrow$ Ağır sönüm

② $\omega_n = \sqrt{\frac{k}{m}}$

③ $\lambda_1 = -\zeta \omega_n - \omega_n \sqrt{\zeta^2 - 1}$
 $\lambda_2 = -\zeta \omega_n + \omega_n \sqrt{\zeta^2 - 1}$

④ Başlangıç Koşulları

$x_0 = A_1 + A_2$
 $\dot{x}_0 = 0 = A_1 \lambda_1 + A_2 \lambda_2$

⑤ $x(t) = A_1 \cdot e^{\lambda_1 t} + A_2 \cdot e^{\lambda_2 t}$

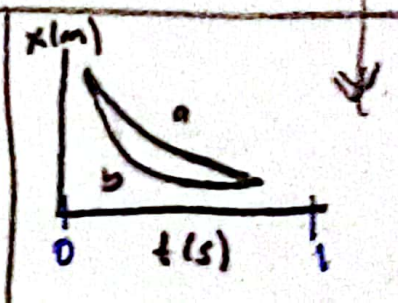
3. ($\zeta = 1$) Kritik Sönüm

① $\zeta = \frac{c}{2\sqrt{km}} = 1 \rightarrow$

② $\omega_n = \sqrt{\frac{k}{m}}$

③ $\lambda_1 = \lambda_2 = -\omega_n$

④ $x(t) = (A_1 + A_2 t) e^{-\omega_n t}$ ⑤ Başlangıç koşulları $\rightarrow A_1 = \dots$
 $A_2 = \dots$



$$\textcircled{1} \quad \gamma = \frac{c}{2\sqrt{km}}$$



$$\gamma < 1 \rightarrow \omega_d$$

$$\textcircled{2} \quad \omega_n = \sqrt{\frac{k}{m}}$$

$$\textcircled{3} \quad \omega_d = \omega_n \sqrt{1-\gamma^2}$$

$$\textcircled{4} \quad \bar{A} = \sqrt{\frac{(\dot{x}_0 + \gamma \omega_n x_0)^2 + (x_0 \omega_d)^2}{\omega_d^2}}$$

$$\phi = \tan^{-1}\left(\frac{x_0 \omega_d}{\dot{x}_0 + \gamma \omega_n x_0}\right)$$

$$\textcircled{5} \quad x(t) = \bar{A} \cdot e^{-\gamma \omega_n t} \cdot \sin(\omega_d t + \phi)$$

$$\gamma > 1 \rightarrow \lambda$$

$$\textcircled{2} \quad \omega_n = \sqrt{\frac{k}{m}}$$

$$\textcircled{3} \quad \lambda_1 = -\gamma \omega_n - \omega_n \sqrt{\gamma^2 - 1}$$

$$\lambda_2 = -\gamma \omega_n + \omega_n \sqrt{\gamma^2 - 1}$$

$$\textcircled{4} \quad \begin{cases} x_0 = A_1 + A_2 \\ \dot{x}_0 = 0 = A_1 \lambda_1 + A_2 \lambda_2 \end{cases} \quad \begin{cases} A_1 = - \\ A_2 = - \end{cases}$$

$$\textcircled{5} \quad x(t) = A_1 \cdot e^{\lambda_1 t} + A_2 \cdot e^{\lambda_2 t}$$

$$\gamma = 1$$

$$\textcircled{2} \quad \omega_n = \sqrt{\frac{k}{m}}$$

$$\textcircled{3} \quad \lambda_1 = \lambda_2 = -\omega_n$$

$$\textcircled{4} \quad x(t) = (A_1 + A_2 t) \cdot e^{-\omega_n t}$$

$$\textcircled{5} \quad \begin{cases} x_0 = \dots \\ \dot{x}_0 = \dots \end{cases} \quad \begin{cases} A_1 = \dots \\ A_2 = \dots \end{cases}$$

$$\dot{x}(t) = (A_2 \cdot e^{-\omega_n t}) + [-\omega_n \cdot e^{-\omega_n t} \cdot (A_1 + A_2 t)]$$

$$(f \cdot g)' = f' \cdot g + g \cdot f$$

...../...../.....

→ Rezonans: Salınan bir sistemde uyarıcı frekansın sistemin doğal titreşim frekansına eşit olduğu durumda gelişen max. değeri olduğu durum.

→ Sınırlı Serbest Titreşim hareketi, sönüm faktörü (γ)'nın olduğu değere göre üç farklı durumda incelenir.

$\gamma < 1$ → Zayıf sönüm

$\gamma = 1$ → Kritik sönüm

$\gamma > 1$ → Ağır sönüm

$$\gamma = \frac{c}{2\sqrt{km}}$$

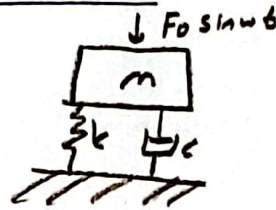
→ Doğal frekans: Bir salınan sistemin serbest durumda titreşim eğiliminde olduğu frekans.
(f_n) ω_n → doğal doğal frekans

→ Doğal periyot: Doğal frekansa bir salınan hareketi için geçen süre.

→ Serbestlik Derecesi: Bir sistemin hareketi bir t anında hareketi belirlemek için gerekli minimum bağımsız koordinat sayısı.

~ ZORLANMIŞ TITREŞİMLER ~

Harmonik Kuvvet



$$m\ddot{x} + c\dot{x} + kx = F_0 \sin wt$$

$$\frac{X}{\frac{F_0}{k}} = \frac{1}{\sqrt{[2\zeta r]^2 + [1 - r^2]^2}}$$

$$r = \frac{\omega}{\omega_n}$$

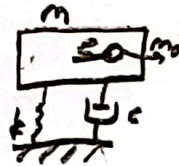
$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\frac{1}{\omega} \frac{d\omega}{dt} = 0.10492 \text{ rad/s}$$

$$\frac{d\omega}{dt} \cdot \frac{1 \text{ dk}}{60 \text{ s}} \cdot \frac{2\pi \text{ rad}}{1 \text{ dev}} = \frac{\text{rad/s}}{c}$$

Kettle Dengesizliği

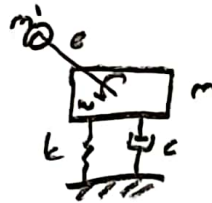
$$m\ddot{x} + c\dot{x} + kx = m_0 \cdot e \cdot \omega^2 \sin wt$$



$$\frac{X}{\frac{m_0 e}{m}} = \frac{r^2}{\sqrt{(2\zeta r)^2 + (1 - r^2)^2}}$$

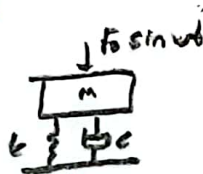
Dönen Dengesizlik Kettle

$$m\ddot{x} + c\dot{x} + kx = m' e \omega^2 \cos wt$$



$$X = \frac{m' e \omega^2}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

Zemine İhtilak Kuvveti



$$\frac{X}{\frac{F_0}{k}} = \frac{1}{\sqrt{(2\zeta r)^2 + (1 - r^2)^2}}$$

$$T = \frac{F_0}{F} = \frac{1 + (2\zeta r)^2}{(2\zeta r)^2 + (1 - r^2)^2}$$

→ Kuvvetin zemin ile oranı

$$\frac{1}{2} m l^2 \ddot{\theta} + c(l\dot{\theta}) + k(l\theta) = \frac{1}{2} 0 \omega^2 \cos wt$$

$$\frac{1}{2} m l^2$$

$$\frac{1}{12} m l^2$$

$$0 \frac{1}{2} m l^2$$

~ RAYLEIGH METODU ~

$$V = \frac{1}{2} \int_0^L EI(x) \left(\frac{\partial^2 y}{\partial x^2} \right)^2 dx$$

$$V = \frac{1}{2} E_0^2 \int_0^L EI(x) [\psi''(x)]^2 dx$$

} F1 değişkenleri
ayrılır

$$T = \frac{1}{2} \int_0^L m(x) \cdot \left(\frac{dy}{dt} \right)^2 dx$$

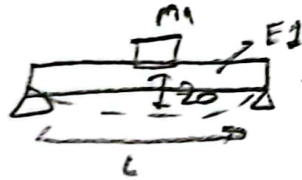
$$T = \frac{1}{2} E_0^2 \omega^2 \cdot \int_0^L m(x) \cdot [\psi(x)]^2 dx$$

} Simetri ve ortada
çok varsa
ayrılır + $m_1 \cdot \psi(x=...)$

• $\int_0^L \sin^2 x = \int_0^L \frac{1}{2} \left(1 - \frac{\cos 2x}{2} \right) \rightarrow \frac{1}{2} \left[x - \frac{\sin 2x}{2} \cdot \frac{1}{2} \right] \Big|_0^L$ | $\int_0^L \cos^2 x dx = \int_0^L \frac{1}{2} [1 + \cos 2x] dx$
= $\frac{1}{2} \left[x + \frac{\sin 2x}{2} \right] \Big|_0^L$

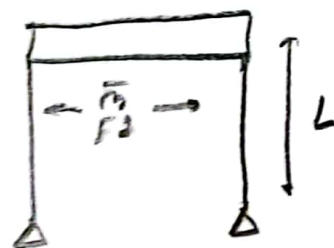
• $T = \frac{1}{2} \int_0^L \bar{m} \cdot V(x) \cdot \cos \omega t dx \rightarrow T_{max} = \frac{1}{2} \int_0^L \bar{m} \cdot V(x) dx$

max için $\cos \omega t = 1$ olur !!



$V \rightarrow 2 \int_0^{L/2} \dots$

$T \rightarrow \left[2 \cdot \int_0^{L/2} \dots + m_1 \cdot (\psi(L/2))^2 \right]$



$V \rightarrow 2 \cdot \int_0^L \dots$

$T \rightarrow \left[2 \cdot \int_0^L \dots + m_1 \cdot (\psi(L))^2 \right]$

$$T = \frac{2\pi}{\omega}$$

$\sin' \rightarrow \cos$ | $\int \sin \rightarrow -\cos$
 $\cos' \rightarrow -\sin$ | $\int \cos \rightarrow \sin$

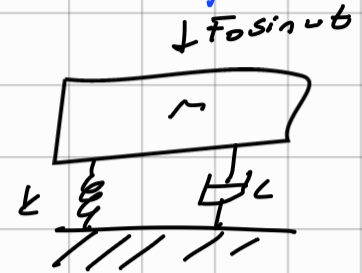
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~ ZORLANMIŞ TİTREŞİMLER ~

Harmonik Kuvvetle Zorlanmış Titreşimler:

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t$$

kuvvetin frekansı ($\omega_n = \omega \rightarrow$ REZONANS)

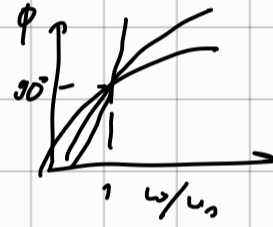
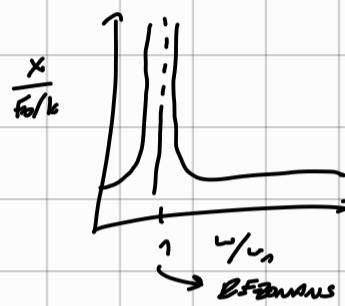


$$\ddot{x} + 2\zeta \omega_n \dot{x} + \omega_n^2 x = \frac{F_0}{m} \sin \omega t$$

$$\zeta = \frac{c}{2m\omega_n} \quad \omega_n = \sqrt{\frac{k}{m}}$$

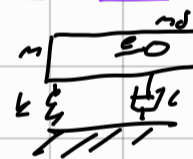
$$\frac{x}{\frac{F_0}{k}} = \frac{1}{\sqrt{[2\zeta(\frac{\omega}{\omega_n})]^2 + [1 - (\frac{\omega}{\omega_n})^2]^2}}$$

$$\phi = \tan^{-1} \left(\frac{2\zeta(\frac{\omega}{\omega_n})}{1 - (\frac{\omega}{\omega_n})^2} \right)$$



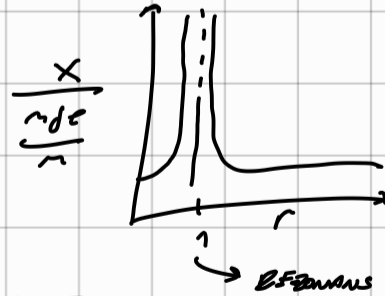
Katı Dengeleyişinden Kaynaklanan Titreşimler:

$$m\ddot{x} + c\dot{x} + kx = m_d e \omega^2 \sin \omega t$$



$$\ddot{x} + 2\zeta \omega_n \dot{x} + \omega_n^2 x = \frac{m_d e \omega^2}{m} \sin \omega t$$

$$\frac{x}{\frac{m_d e}{m}} = \frac{r^2}{\sqrt{[2\zeta r]^2 + [1 - r^2]^2}}$$



$r = \frac{\omega}{\omega_n}$
estherki zori!

Zemin İstikrarı Kuvveti:

$$F_{cr} = \sqrt{(kX)^2 + (cX\omega)^2} = kX \sqrt{1 + (2\zeta r)^2}$$



$$\frac{x}{\frac{F_0}{k}} = \frac{1}{\sqrt{[2\zeta r]^2 + [1 - r^2]^2}}$$

$$T = \frac{F_{cr}}{F} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{[2\zeta r]^2 + [1 - r^2]^2}}$$

kuvvetin zemindirimi oranı

Dönen Dengelemenig Kütüklerin Kol Ağırlığı Zitirosinik:

$$m\ddot{x} + c\dot{x} + kx = \underbrace{m'e\omega^2 \cos \omega t}_{F_0}$$

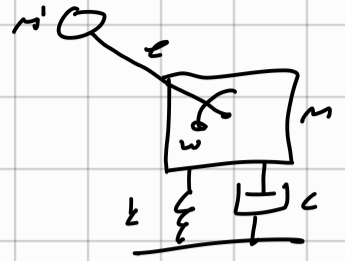
$$X = \frac{m'e\omega^2}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}}$$

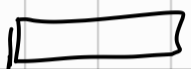
$$\Theta_0 = \frac{F_0}{\sqrt{(k-m)^2 + (c\omega)^2}}$$

$$\phi = \tan^{-1} \frac{c\omega}{k-m\omega^2}$$

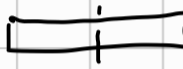
$$\phi = \tan^{-1} \frac{c\omega}{k-m\omega^2}$$

$$\Theta(t) = \Theta_0 \cos(\omega t - \phi)$$





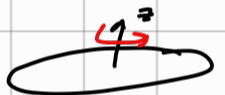
$$I = \frac{1}{3} m l^2$$



$$I = \frac{1}{12} m l^2$$

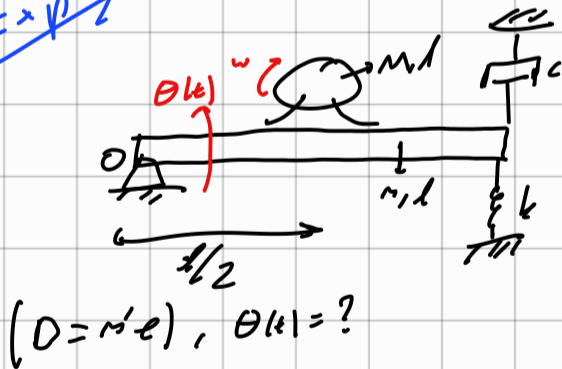


$$I = \frac{1}{2} m r^2$$



$$I = \frac{1}{2} m r^2$$

~~F x p~~



$$\left(\frac{1}{3} m l^2 + \frac{1}{2} M l^2 \right) \ddot{\Theta} + \underbrace{c(l\dot{\Theta})}_{?} + \underbrace{k(l\Theta)}_{?} = \underbrace{\frac{1}{2} D \omega^2 \cos \omega t}_{?}$$

$$= \left(\frac{m}{3} + \frac{M}{4} \right) \ddot{\Theta} + c\dot{\Theta} + k\Theta = \frac{D \omega^2}{2l} \cos \omega t$$

$$M = \frac{m}{3} + \frac{M}{4} = \underline{6 \text{ kg}}$$

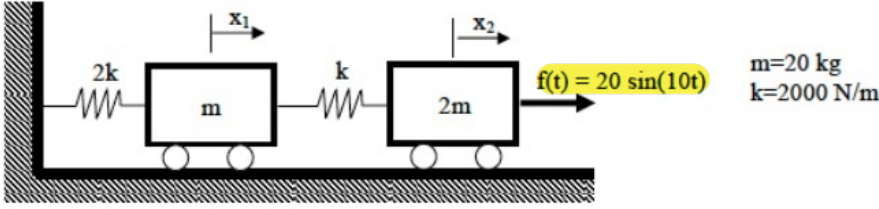
$$F_0 = \frac{D \omega^2}{2l} = \underline{12}$$

$$\Theta_0 = \frac{F_0}{\sqrt{(k-m)^2 + (c\omega)^2}} = \underline{0.022 \text{ rad}}$$

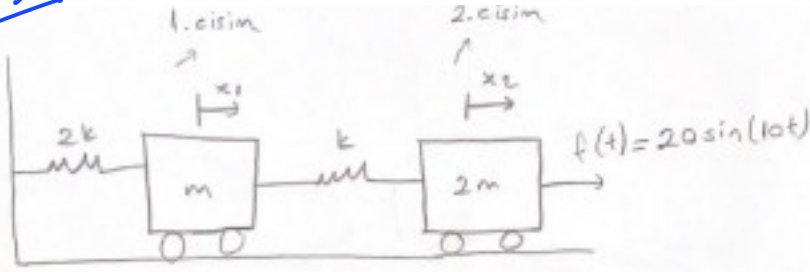
$$\phi = \tan^{-1} \frac{c\omega}{k-m\omega^2} = \underline{1.068 \text{ rad}}$$

$$\Theta(t) = \Theta_0 \cos(\omega t - \phi) = \boxed{0.022 \cos(10t - 1.068) \text{ rad}}$$

Şekilde verilen iki serbestlik dereceli sistemin gösterilen şekildeki harmonik zorlama altında x_1 ve x_2 yer değiştirme genliklerini hesaplayınız.



① FBD



$m = 20 \text{ kg}$
 $k = 2000 \text{ N/m}$

Yay Enerjisi $\rightarrow \frac{1}{2} 2k x_1^2 + \frac{1}{2} k (x_2 - x_1)^2 = \text{Yay}$

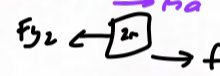
$\frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} 2m \dot{x}_2^2$



$m\ddot{x} = -2kx_1 + k(x_2 - x_1)$

$m\ddot{x} + 2kx_1 + kx_1 - kx_2$

$m\ddot{x} + 3kx_1 - kx_2 = 0$ (1. cisim için)



$2m\ddot{x} = f - k(x_2 - x_1)$

$2m\ddot{x} + kx_2 - kx_1 = f$ (2. cisim için)

0.43 rkr
yaz

② $m\ddot{x}_1 + 3kx_1 - kx_2 = 0$ (1. cisim için)

$2m\ddot{x}_2 + kx_2 - kx_1 = f$ (2. cisim için)

$\rightarrow 20\ddot{x}_1 + 6000x_1 - 2000x_2 = 0$ (A)

$\rightarrow 40\ddot{x}_2 + 2000x_2 - 2000x_1 = 20\sin(10t)$ (B)

③ yer değiştirme genliklerini fonksiyon olarak yazarsak;

$x_1(t) = X_1 \sin(10t) \rightarrow 2 \text{ kere türevi alır.} \rightarrow \ddot{x}_1(t) = -100 X_1 \sin(10t)$

$x_2(t) = X_2 \sin(10t) \rightarrow 2 \text{ kere türevi alır.} \rightarrow \ddot{x}_2(t) = -100 X_2 \sin(10t)$

A Denklemini
 $\rightarrow 20(-100 X_1 \sin(10t)) + 6000(X_1 \sin(10t)) - 2000(X_2 \sin(10t)) = 0$

$-2000 X_1 + 6000 X_1 - 2000 X_2 = 0 \Rightarrow 4000 X_1 - 2000 X_2 = 0$

B Denklemini
 $\rightarrow 40(-100 X_2 \sin(10t)) + 2000(X_2 \sin(10t)) - 2000(X_1 \sin(10t)) = 20 \sin(10t)$

$-4000 X_2 + 2000 X_2 - 2000 X_1 = 20$

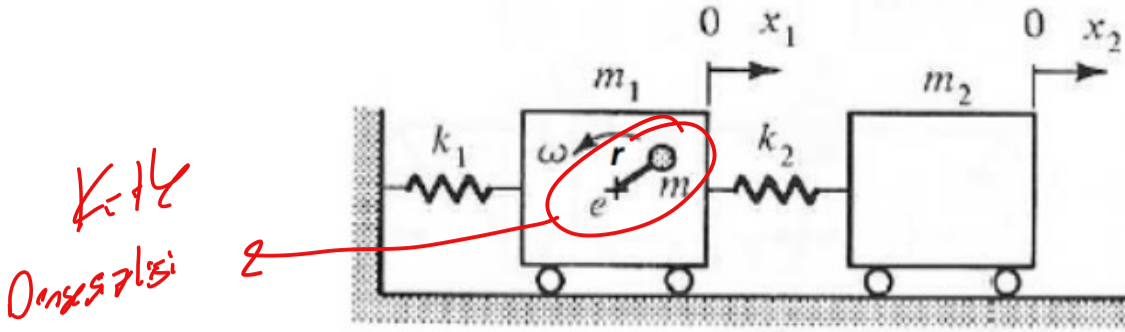
$-2000 X_2 - 2000 X_1 = 20$

A $\rightarrow 2X_1 = X_2$

B $\rightarrow -100 X_2 - 100 X_1 = 1$

$X_1 = -0.0033 \text{ m}$
 $X_2 = 2(-0.0033) = -0.0066 \text{ m}$

2. Şekilde verilen iki serbestlik dereceli sistemin gösterilen şekildeki **harmonik zorlama** altında ve sistemin kararlı durumundaki x_1 ve x_2 yer değiştirme genliklerini **hesaplayınız.** ($m_1 = m_2 = 20 \text{ kg}$; $m = 1 \text{ kg}$; $r = 1 \text{ m}$; $k_1 = 2000 \text{ N/m}$; $k_2 = 1000 \text{ N/m}$; $\omega = 5 \text{ rad/s}$) (35 puan)



- ① $\rightarrow m_1 a$ $\rightarrow m_2 a$
- $k_1 x_1 \leftarrow m_1 \leftarrow k_2(x_2 - x_1)$ $k_2(x_2 - x_1) \rightarrow m_2$
- ② $m_1 \ddot{x}_1 = m' e \omega^2 \cos \omega t - k_1 x_1 - k_2 x_2 + k_2 x_1$
- $m_1 \ddot{x}_1 + k_1 x_1 = m' e \omega^2 \sin \omega t \rightarrow 20 \ddot{x}_1 + 2000 x_1 = 1 \cdot 1 \cdot 5^2 \sin 5t$
- $m_2 \ddot{x}_2 = k_2(x_2 - x_1) \rightarrow m_2 \ddot{x}_2 - k_2 x_2 + k_2 x_1 = 0 \rightarrow 20 \ddot{x}_2 - 1000 x_2 + 1000 x_1 = 0$
- ③ $x_1(t) = A \cdot \sin 5t \rightarrow \ddot{x}_1(t) = -25 \cdot A \cdot \sin 5t$
- $x_2(t) = B \cdot \sin 5t \rightarrow \ddot{x}_2(t) = -25 \cdot B \cdot \sin 5t$
- ④ $20(-25A \sin 5t) + 2000(A \sin 5t) = 25 \cdot \sin 5t \rightarrow A \sin 5t (-500 + 2000) = 25 \sin 5t$
- $A = 0.0167$
- $20(-25B \sin 5t) - 1000(B \sin 5t) + 1000(A \sin 5t) = 0$
- $-1500 B + 1000 A = 0$
- $\rightarrow B = \frac{-1000 A}{-1500} \rightarrow B = 0.0111$

~ RAYLEIGH METODU ~

$$\int_0^L \sin^2 x = \int_0^L \frac{1}{2} \left(1 - \frac{\cos 2x}{2} \right) dx$$

$$= \frac{1}{2} \left[x - \frac{\sin 2x}{2} \cdot \frac{1}{2} \right]_0^L$$

$$v(x,t) = \underbrace{\psi(x)}_{\text{Göçlü}} \underbrace{z_0 \sin \omega t}_{\text{Titreşim formu fakt.}}$$

$$V = \frac{1}{2} \int_0^L EI(x) \left(\frac{\partial^2 v}{\partial x^2} \right)^2 dx \rightarrow V_{max} = \frac{1}{2} z_0^2 \int_0^L EI(x) [\psi''(x)]^2 dx$$

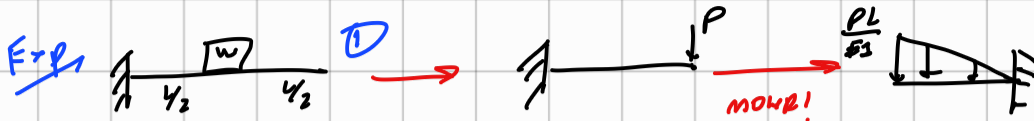
~ toplam şilil deşitilme eşitili ~

$$T = \frac{1}{2} \int_0^L m(x) [\dot{v}(x,t)]^2 dx \rightarrow T_{max} = \frac{1}{2} z_0^2 \omega^2 \int_0^L m(x) [\psi'(x)]^2 dx$$

$$T_{max} = V_{max} \rightarrow \omega^2 = \frac{\int_0^L EI(x) [\psi''(x)]^2 dx}{\int_0^L m(x) [\psi'(x)]^2 dx}$$

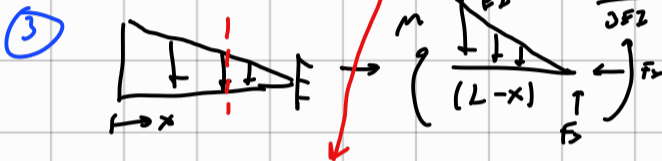
VEGA

$$V_{max} = \frac{1}{2} g z_0^2 \int_0^L m(x) \cdot \psi(x) dx, T_{max} \approx \rightarrow \omega^2 = g \cdot \frac{\int_0^L m(x) \cdot v(x) dx}{\int_0^L m(x) \cdot [\psi'(x)]^2 dx}$$



$$z_0 = \frac{PL}{EI} \cdot \frac{L}{2} \cdot \frac{2L}{3} = \frac{PL^3}{3EI}$$

A



$$\frac{PL^3}{3EI} - M + \frac{Px}{EI} \cdot \frac{L-x}{2} \cdot \frac{2(L-x)}{3} = 0$$

$$M = \frac{PL^3}{3EI} + \frac{PxL - Px^2}{2EI} \cdot \frac{2(L-x)}{3}$$

$$\frac{PxL^2 - Px^2L - Px^2L + Px^3}{3EI}$$

$$z_0 \psi(x) = \frac{PL^3}{3EI} \left[\frac{3x^2L - x^3}{2L} \right]$$

$$M = \left[\frac{3x^2L - x^3}{2L} \right] ??$$

$$V_{max} = \frac{1}{2} P z_0 = \frac{1}{2} \frac{3EI}{L^3} z_0^2$$

$$z_0 = \frac{PL^3}{3EI} \rightarrow P = \frac{3EI z_0}{L^3}$$

$$T_{max} = V_{max}$$

$$T_{B_{max}} = \frac{\bar{m}}{2} \omega^2 z_0^2 \int_0^L \psi(x)^2 dx = \frac{33}{160} \frac{\bar{m} L}{2} \omega^2 z_0^2$$

$$T_{W_{max}} = \frac{1}{2} m \omega^2 \left[v(x = \frac{L}{2}) \right]^2 = \frac{25}{256} \frac{m}{L} \omega^2 z_0^2$$

$$\omega^2 = -$$

Ex 2

Sondaj gerisi için titreşim formu

$$v(x) = \frac{v_0}{2} \left[3 \left(\frac{x}{L} \right)^2 - \left(\frac{x}{L} \right)^3 \right]$$

$$\bar{m} = 9000 \text{ kg}, EI = 6.22 \cdot 10^9 \text{ Nm}^2, L = 100 \text{ m}$$

$$v(x,t) = v(x) \cdot \sin \omega t$$

① $\frac{dv}{dt} = \omega^2 \cdot v(x) \cdot \cos \omega t$

$$\frac{dv}{dx} = \frac{v_0}{2} \left[\frac{6x}{L^2} - \frac{3x^2}{L^3} \right] \rightarrow \frac{d^2v}{dx^2} = \frac{v_0}{2} \left[\frac{6}{L^2} - \frac{6x}{L^3} \right]$$

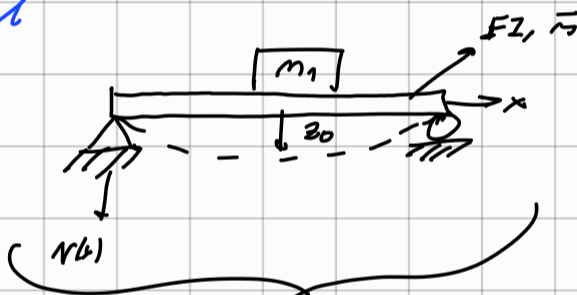
② $V_{max} = \frac{1}{2} EI \int_0^L \left(\frac{d^2v}{dx^2} \right)^2 dx = \frac{1}{2} EI \int_0^L \left(\frac{v_0}{2} \left[\frac{6}{L^2} - \frac{6x}{L^3} \right] \right)^2 dx = \frac{3EI}{2L^3} v_0^2$

$T_{max} = \frac{1}{2} \int_0^L m(x) \left[\dot{v}(x,t) \right]^2 dx = \frac{1}{2} \bar{m} \int_0^L \omega^2 v(x)^2 \cos^2 \omega t dx = \frac{1}{2} \omega^2 v_0^2 \frac{33 \bar{m} L}{140}$

max için $\cos^2 \omega t = 1$ alır

③ $T_{max} = V_{max} \rightarrow \frac{3EI}{2L^3} v_0^2 = \frac{1}{2} \omega^2 v_0^2 \frac{33 \bar{m} L}{140} \rightarrow \boxed{\omega = 2.825 \text{ rad/s}}$

Ex 3



$$v(x) = \frac{px(3x^2 - 4x^2)}{48EI} = \frac{pxL^2}{48EI} \left[3 - 4 \left(\frac{x}{L} \right)^2 \right] = 20 \psi(x) \quad 0 \leq x \leq \frac{L}{2}$$

Titrşim şeklin 2 duruma göre bulunur.

Deplasman formu biris ortama ($x = \frac{L}{2}$) göre simetrik

- a) $m_1 = 0$
- b) $m_1 = 3 \bar{m} L$

① $z_0 = \frac{pL^3}{48EI}$, $\psi(x) = \frac{x}{L} \left[3 - 4 \left(\frac{x}{L} \right)^2 \right] \rightarrow \psi(x)' = \frac{1}{L} \left(3 - 4 \left(\frac{x}{L} \right)^2 \right) + \frac{x}{L} \left(-\frac{8x}{L^2} \right)$

② $\psi(x)'' = \frac{-8x}{L^2} - \frac{16x}{L^3} = \frac{-24x}{L^3}$

$V_{max} = \frac{1}{2} z_0^2 \int_0^L EI \left[\psi(x)'' \right]^2 dx \rightarrow V_{max} = \frac{1}{2} z_0^2 \cdot 2 \cdot \int_0^{L/2} EI \left(\psi(x)'' \right)^2 dx = 20^2 EI \int_0^{L/2} \frac{576 x^2}{L^6} dx = z_0^2 \frac{24EI}{L^3}$

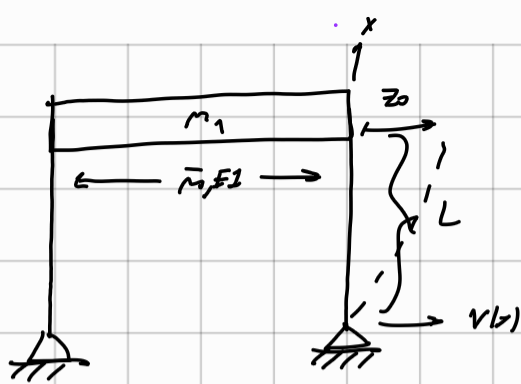
$T_{max} = \frac{1}{2} z_0^2 \omega^2 \int_0^L m(x) \left[\psi(x) \right]^2 dx \rightarrow T_{max} = \frac{1}{2} z_0^2 \omega^2 \left(2 \cdot \int_0^{L/2} \bar{m} \left[\frac{x}{L} \left(3 - 4 \left(\frac{x}{L} \right)^2 \right) \right]^2 dx + m_1 \left[\frac{L/2}{L} \left(3 - 4 \left(\frac{L/2}{L} \right)^2 \right)^2 \right] \right)$

$T_{max} = \frac{1}{2} z_0^2 \omega^2 \left(2 \bar{m} \int_0^{L/2} \left[3 \left(\frac{x}{L} \right)^2 - 24 \left(\frac{x}{L} \right)^4 + 16 \left(\frac{x}{L} \right)^6 \right] dx + m_1 \left(\frac{1}{4} \right)^2 \right) = \frac{1}{2} z_0^2 \omega^2 \left(\frac{13}{35} \bar{m} L + m_1 \right)$

③ $V_{max} = T_{max} \rightarrow \omega^2 = \frac{48EI}{\left(\frac{13}{35} \bar{m} L + m_1 \right) L^3}$, $T = \frac{2\pi}{\omega} \rightarrow \boxed{T = \frac{2}{\omega} \sqrt{\frac{13 \bar{m} L^4}{105 EI}}}$

$m = 0 \rightarrow \bar{m} L \rightarrow \boxed{T = \frac{\pi}{2} \sqrt{\frac{122 \bar{m} L^4}{105 EI}}}$

F1/5



$$v(x) = \frac{\rho(3L^2x - x^3)}{12F_2} = \frac{\rho L^3}{12F_2} \left[3\frac{x}{L} - \left(\frac{x}{L}\right)^3 \right]$$

① $z_0 = \frac{\rho L^3}{12F_2}$, $\psi(x) = 3\frac{x}{L} - \left(\frac{x}{L}\right)^3 \rightarrow \psi(x)'' = -\frac{6x}{L^2}$

② $V_{max} = \frac{1}{2} z_0^2 \int_0^L F_2(x) [\psi(x)']^2 dx \rightarrow \frac{1}{2} z_0^2 \cdot 2 \int_0^L F_2 \left(-\frac{6x}{L^2}\right)^2 dx = z_0^2 F_2 \int_0^L 36 \frac{x^2}{L^4} dx = z_0^2 \frac{12F_2}{L^3}$

2 x 6 x 6

$T_{max} = \frac{1}{2} z_0^2 \omega^2 \int_0^L m(x) [\psi(x)]^2 dx \rightarrow \frac{1}{2} z_0^2 \omega^2 \left[2 \int_0^L \bar{m} \left[3\frac{x}{L} - \left(\frac{x}{L}\right)^3 \right]^2 dx + m_1 \left[3\frac{L}{L} - \left(\frac{L}{L}\right)^3 \right]^2 \right]$

birisi 52, 501 birisi

③ $V_{max} = T_{max}$

$\omega^2 = \frac{6F_2}{L^3 \left(\frac{15}{55} \bar{m} L + m_1 \right)}$

$\tau = 2\pi \sqrt{\frac{L^3 \left(\frac{15}{55} \bar{m} L + m_1 \right)}{6F_2}}$